EC5555. Problem Set 1: Vectors & Matrices

Reading. Chiang and Wainwright, chapter 4

1. Write the following in vector form

i. p_1 is £8, p_2 is £1.5.
ii. p_1 is £4, p_2 is £0.5 and p_3 is £12
iii. Anne buys four units of good 1, 6 units of good 2 and nine units of good 3.
iv. The height of the building is z, the width is 12 and the depth is x

2. What are the dimensions of the following matrices and vectors?

i. $\begin{pmatrix} 1 & 0 \\ 4 & 0 \\ 2 & 1 \end{pmatrix}$ ii. $\begin{pmatrix} 3 & 6 & 1 \\ 2 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ iii. $\begin{pmatrix} 2 & 7 & 7 \\ 0 & x & 6 \end{pmatrix}$ iv. $\begin{pmatrix} 1 & 0 & 0 & 2 & 1 \end{pmatrix}$

v. (1)

3. Plot the following vectors:

i. x = (4 0) ii. y = (3 1) iii. z = (2 2)

4. Using the vectors from question 3, find and plot

x + y
x – y
and
2z + y

5. In each case find ax where:

i. x = (4 0) and a = 2 ii. x = (3 1) and a = 4 iii. x = (2 2) and a = b
iv. x = (4 1) and a = 0.

6. Show that a, b, and c are not linearly independent

i. a = (4 0) b = (2 0) c = (1 1)
ii. a = (6 0) b = (3 4) c = (3 2)
iii. a = (4 1 1) b = (2 3 0) c = (0 -5 1)

7. Show that a and b are linearly independent.

i. a = (4 0) b = (1 1)
ii. a = (4 1 1) b = (2 3 0)

8. Find a.b (the dot product) in each case

i. a = (4 0)   b = (1 1)
ii. a = (4 1 1) b = (2 3 0)

9. You have three 1xn vectors, a b and c
Will the following two operations produce the same result?

i. Find a.b then multiply c by the resulting scalar.
ii. Find b.c then multiply a by the resulting scalar.

10. Consider the following matrices

i. $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 2 & 1 \end{pmatrix}$ ii. B= $\begin{pmatrix} 1 & 0 & 1 \\ -2 & 2 & 1 \\ 0 & 2 & 0 \end{pmatrix}$ iii. $C = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & -1 \end{pmatrix}$ iv. $D = \begin{pmatrix} 1 & 0 & 2 & 2 & 1 \end{pmatrix}$

Which pairs can you multiply?
(note: AB is not the same as BA etc. so you need to consider AB and BA separately).

11. Given

$A = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ B= $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}$ $C = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & -1 \end{pmatrix}$ $D = \begin{pmatrix} 1 & 0 & 2 & 2 & 1 \end{pmatrix}$

Find i. BA ii. BB’ iii. CB and iv. AD

12. Let $A = \begin{pmatrix} 1 & 1 \\ -2 & 0 \end{pmatrix}$; $B = \begin{pmatrix} 0 & 4 \\ -2 & 0 \end{pmatrix}$ $C = \begin{pmatrix} -2 & 2 \\ 3 & 1 \end{pmatrix}$ $D = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 4 & 0 \\ 1 & 2 & 3 \end{bmatrix}$

Find
i. det. A   ii. det. B.   iii. det. A’   iv. det. C.   v. det. D

13. Find the trace of B, the trace of B’ and the trace of A +B
What do you find?

14. Write these systems of equations in matrix form

i. $x_1 + x_2 = 2$; $x_1 – 3x_2 = 4$ ii. $x_1 + x_2 + x_3 = 1$; $x_1 – x_2 = 4$; $x_2 + x_3 = 7$.

Example:

$3x_1 = 7$; $2x_1 + 6x_2 = 0$ becomes:

$$\begin{pmatrix} 3 & 0 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$$

15. Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ -2 & 0 & 1 \\ 0 & 4 & 0 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 4 & 0 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad C = \begin{pmatrix} -2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 4 & 0 \end{pmatrix} \quad D = \begin{pmatrix} -4 & 10 & 1 \\ 1 & 5 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Find

i. det. A ii. det. B. iii. det. A' and iv. det. C

v. What do you conclude about the relationship between the determinant of a matrix and the determinant of the transpose of the same matrix?

vi. What is the the effect of swapping rows on the determinant of a matrix?

vii. the determinant of D?

16. Do the following have a unique solution?

i. $\begin{pmatrix} 3 & 0 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$ ii. $\begin{pmatrix} 3 & 9 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ iii. $\begin{pmatrix} 1 & 1 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$

17. Let

$$A = \begin{pmatrix} 1 & 1 \\ -2 & 0 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 4 \\ -2 & 0 \end{pmatrix} \quad C = \begin{pmatrix} -2 & 2 \\ 3 & 1 \end{pmatrix}$$

Find the inverses of any non-singular matrices in (you may need to look back in your notes for the definition of non-singular)

18. Let

$$A = \begin{pmatrix} -4 & 10 & 1 \\ 1 & 5 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 6 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

What is the rank of A? What is the rank of B?

19. Find the inverses of the following matrices

$$A = \begin{pmatrix} 1 & 1 & 1 \\ -2 & 0 & 1 \\ 0 & 4 & 0 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 4 & 0 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad C = \begin{pmatrix} -2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 4 & 0 \end{pmatrix}$$

20. Find the solution for x_1 x_2 and x_3

i. $\begin{pmatrix} 4 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 2 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$ ii. $\begin{pmatrix} 3 & 1 & -1 \\ 2 & 0 & 2 \\ 4 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ iii. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

21. Use Cramer's rule to find x_2 in the following system of equations

$$\begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$$

22. Use Cramer's rule to find consumption (C) in i. and x_2 in ii.
What happens to consumption if exports (X) rise marginally?

i. $\begin{pmatrix} 1 & -b & 0 & 0 \\ -1 & 1 & -1 & -1 \\ 0 & 1 & 0 & -m \\ 0 & 0 & 0 & -d \end{pmatrix}\begin{pmatrix} C \\ Y \\ M \\ I_1 \end{pmatrix} = \begin{pmatrix} a \\ X + G + I_0 \\ 0 \\ r \end{pmatrix}$ ii. $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

23. Given a markov model of labour force dynamics

$$x_t = Mx_{t-1}$$

where $x_0 = \begin{bmatrix} employed_0 \\ unemployed_0 \end{bmatrix} = \begin{bmatrix} 900 \\ 100 \end{bmatrix}$ $\quad M_t = \begin{bmatrix} ee & ue \\ eu & uu \end{bmatrix} = \begin{bmatrix} .95 & .30 \\ .05 & .70 \end{bmatrix}$

Find the 3rd period levels of employment and unemployment as the model converges towards its steady state values

In a steady state these flows should be equal. What are the implied flows for this 3rd order round?