Problem set 5 - Constrained Optimization

1)

Consider the following problem

$$\max_{\{K,L\}} aK + \ln L$$
$$\text{subject to } K + bL = M$$
$$\text{where a>0, b>0, M >0}$$

1) Write the Lagrangian
2) Write the first order conditions
3) Find the quantities of K and L that satisfy the first order conditions
4) Write the bordered Hessian matrix
5) Prove that this result is a local maximum (second order condition)
6) Prove that this result is a global maximum
7) Find the marginal effect of a change of parameter b on the maximized value
8) Find the marginal effect of a change of parameter *a* on the maximized value
9) Find the marginal effect of a change of parameter M on the maximized value

2)

Define the function *f* by $f(x, r) = x^{1/2} - rx$, where $x \geq 0$. On a graph with *r* on the horizontal axis, sketch the function for several values of *x (for example x=o.5, x=1, x=2)*. Sketch, in addition, the value function *f* *, where *f* *(*r*) is the maximal value of *f* (*x*, *r*) for each given value of *r*.

3)

a) Write the KT conditions of the following problem:

$$\max_{\{x,y\}} a \cdot (x \cdot y)^b$$
$$s.t. \quad 100 - \mathrm{m} \cdot x - n \cdot y \geq 0$$
$$x \leq 4$$
$$y \geq 0$$
$$\text{where } b < 0.5 \; a < 0$$

b) Write the KT conditions of the modified lagrangean

c) Check if these condition are both necessary and sufficient

4)

a) Solve the following problem:

$$\max_{\{x,y\}} (100 - x) \cdot y$$
$$s.t. \quad x \cdot y \geq 10$$
$$x \leq 2$$
$$y \geq 0$$

b) Check if KT condition are both necessary and sufficient