

Problem Set 6

Integration

1. Find the indefinite integral of:

- $x+1$
- $-2x^2$
- xe^x

2. Find the definite integral between 0 and 2 of,

- $2x-2$
- $e^{0.5x}$

3. Which of these improper integrals exists?

- $\int_0^2 x^{-2} dx$.
- $\int_0^{\infty} e^{-2x} dx$

4. Use the substitution method to integrate the following

$$\int_0^2 \frac{3x^2}{(x^3 + 1)^2} dx$$

5. Integrate by parts

$$\int_2^5 \frac{3x}{(x+1)^2} dx$$

6. Given the demand function $P = 42 - 5Q - Q^2$

Find the consumer surplus at the equilibrium price of $p = 6$

7. Given Investment $I(t) = 9t^{0.5}$

Find the level of capital stock in a) after 8 years b) between years 5 and 8

Differential equations.

4. Write these in differential equation form:

- the rate of change of y with respect to time is proportional to time.
- the rate of change of y with respect to time is equal to the difference between y and the time.

5. Which of these differentials is exact?

6. Find the general solution of $dy/dt + 3t^2y = t^2$
(let $v = 3t^2$ and $z = t^2$)

7. Solve the exact nonlinear differential equation

$$F(y,t) = (6yt + 9y^2)dy + (3y^2 - 8t) dt = 0$$

Dynamic optimization

In a simple growth model, the welfare of the single consumer who has a lifetime of T , is given by $\int_0^T 2c^{0.5} e^{-rt} dt$, where c is consumption and r is the discount rate with $r < 0.9$.

In this economy, y = output, i = investment and K = capital stock.

1. Provide an economic interpretation for each of these equations:

- i) $c = y - i$
- ii) $y = K$
- iii) $\dot{K} = i - 0.1K$
- iv) $K \geq 0$.
- v) $K(0) = K_0$

2. Use substitution to get an equation for the evolution of the capital stock that is a function of K and c .

3. Hence set up the Hamiltonian for this problem and find the first order conditions.

4. Find the optimal path for consumption and sketch it.

5. What happens to the optimal capital stock as $t \rightarrow T$?