What Does The Solow Model Tell Us About Economic Growth? : Complete and Partial Cross-country Excludability of Technologies.*

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Abstract: This paper presents, within a framework of the Solow model, evidence that there should be two different reasons for convergence. One is due to diminishing returns to capital and the other is due to technological diffusion. This paper shows that OECD and low income countries follow a pattern of conditional convergence but middle income countries do not. This seems to imply that technological diffusion has a very large effect only on middle income countries because technologies are partially excludable across countries. In other words, technologies are easily diffused in middle income countries but not in low income countries.

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1. INTRODUCTION

There has been controversy over the growth regressions deployed in the neo-classical growth models. This is generally referred to as the 'convergence controversy'. The neo-classical growth models suggest that an economy converges to its own balanced growth path. This, in turn, implies that if we control for the exogenously determined variables such as the population growth rate, the investment rates of physical and human capital, political stability, etc, and assume that all the economies face the same exogenously determined constant growth rate of technology, we should observe conditional convergence. Mankiw, Romer, and Weil (1992) (hereafter MRW) undertook a cross-country regression analysis within the framework of the Solow model and found evidence for conditional convergence by augmenting the Solow model with human capital. Barro and Sala-i-Martin (1992a, 1992b) carried out the convergence tests and conditioned regressions by using regional data on US and Japan, and observed conditional convergence within these two countries.[1]

Despite these findings, many economists criticise the neo-classical growth model and its empirical tests. Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) were not satisfied with the assumption of exogenously determined growth rate of technology and established models which endogenize a country's technology. Temple (1998), and Aghion and Howitt (1998, Ch.1) pointed out an important failure in the neo-classical growth model and argue that 'convergence from above' should not be observed although the Solow model predicts it. (By re-testing MRW's augmented Solow model, Cho and Graham (1996) found out that especially poor countries have been 'converging from above'.) Most importantly, many growth economists argue that the idea of treating technologies as nonrival and non-excludable goods in the neo-classical model is not appropriate.[2] They argue that it is indefensible to assume a constant common growth rate of technology and a common initial level of technology in the cross-country regressions. The levels and growth rates of technologies somehow should differ across economies.

The main purpose of this paper is to re-examine the Solow model by adapting different assumptions about technologies. Instead of assuming non-excludability of technologies, this paper considers two kinds of assumptions: *complete and partial cross-country excludability of technologies*. *Complete cross-country excludability of technologies* implies that although in principle new technologies can be used by everyone at the same time the actual usage of the new technologies are restricted to only those who invented. *Partial cross-country excludability of technologies* implies that some countries can benefit from technological diffusion from technological leader. By relaxing the non-excludability assumption, this paper attempts to show what the conventional analysis in the Solow model tends to miss out and reconsider the validity of the model by applying a new cross-country regression method. Then, considering the obtained results, this paper argues that there should be two different reasons for convergence: diminishing returns to capital and technological diffusion.

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[1] MRW (1992) and Barro and Sala-i-Martin (1992a, 1992b) both concluded that the estimated speed of convergence, β, is about 0.02 per year around the balanced growth path.

[2] Romer (1994) argues that important discoveries are usually excludable for at least some period of time.

Section 2 re-examines the motion of the economy within the framework of the Solow model by using *capital per labour* not *capital per effective labour*. This approach allows us to analyse the effect on the motion of the economy when the model allows the initial level of technologies to differ across countries. Section 3 shows the shortcomings in MRW's cross-country regressions and looks at an alternative approach to test the Solow model. Sections 4 and 5 reveal rather different results about empirical validity of the Solow model from MRW's findings. It is found that the simple Solow model seems to be empirically valid for OECD countries and the countries converging from above their balanced growth paths but invalid for those countries converging from below (not including OECD countries) when we assume *complete cross-country excludability of technologies*. It is also shown that the estimated coefficient for the speed of convergence could be larger than the conventional estimated value without contradicting the Solow model's prediction. Section 6 evaluates the results by introducing the notion of *overshooting the balanced growth path.* Then in order to explain the obtained results from the regression analysis we assume that some countries should benefit due to technological diffusion from a leading country but other countries may not incur such a benefit, - *partial cross-country excludability of technologies* -. In other words, it is shown that the obtained results might imply that the growth rates of technologies are far from constant for some countries although they may be almost constant for other countries.

2. ALTERNATIVE ANLYSIS OF THE SOLOW MODEL

I begin by re-stating the simple Solow model. A conventional way to analyse the motion of economy in the Solow model is to use *capital per effective labour*. However, *capital per labour* is used throughout this paper to analyse the model. This approach allows us to pay greater attention to the level of technology when we analyse the model.

A. Model

A Cobb-Douglas production function case in the Solow (1956) model is considered. The Cobb-Douglas function takes the form of labour-augmenting technological progress. The function at time t is, therefore, given by:

(1) $$Y[t] = K[t]^{a} \, (A[t]L[t])^{1-a} \qquad 0 < \alpha < 1,$$

where Y, K, A, and L denote output, capital, the level of technology, and labour, respectively. The Solow model assumes that population growth rates and technological growth rates are exogenously determined. Thus, the level of technology and the amount of labour at time t are given by:

(2) $$L[t] = L[0]e^{nt}$$

(3) $$A[t] = A[0]e^{gt} ,$$

where $L[0]$ and $A[0]$ are the initial amount of labour and the initial level of technology, respectively. L and A grow at the exogenously determined rates n and g.

Assuming that the rates of saving and depreciation are exogenous and constant, the evolution of capital can be described as:

$$(4) \qquad \dot{K}[t] = sY[t] - dK[t],$$

where $\dot{K}[t]$ denotes differentiation with respect to time, and s and d are the rates of saving and depreciation, respectively.

By defining $k[t]$ as the capital per unit of labour , the evolution of $k[t]$ is given by:

$$(5) \qquad \dot{k}[t] = sA[0]^{1-a} e^{(1-a)gt} k[t]^a - (n+d)k[t].$$

Notice that equation (5) describes the evolution of $K[t]/L[t]$, and not $K[t]/A[t]L[t]$. This method make it possible to capture the impact of differences in the initial level of technology $A[0]$ on the levels and the growth rates of income per unit of labour.

B. Graphical Analysis for Balanced Growth Path

To analyze the model, three cases can be considered: $\dot{k}[t] = 0$, $\dot{k}[t] > 0$, and $\dot{k}[t] < 0$. First, assume $\dot{k}[t] = 0$. Using Equation (5), it implies:

$$(6) \qquad \ln k[t] = \frac{1}{1-a}\ln s - \frac{1}{1-a}\ln(n+d) + \ln A[0] + gt .$$

Equation (6) describe all combinations of t and ln$k[t]$ which give zero growth rate of $k[t]$. Since a , s, n, d, $A[0]$, and g are constant, equation (6) is a linear line in the (t, ln$k[t]$) space. This line is called '*stationary lnk[t] line'*. The line represents the locus of points in the ($\ln k[t]$, t) space where $\dot{k}[t] = 0$ is satisfied at any given point of time. In other words, the stationarity is only local. The slope of this line is g. Next, consider the case: $\dot{k}[t] > 0$. Using equation (5), this case is described by the following equation:

$$(7) \qquad \ln k[t] < \frac{1}{1-a}\ln s - \frac{1}{1-a}\ln(n+d) + \ln A[0] + gt .$$

Equation (7) implies that ln$k[t]$ is below *stationary lnk[t] line* for any given t. Finally consider the case: $\dot{k}[t] < 0$. Using equation (5), this case is described by equation:

$$(8) \qquad \ln k[t] > \frac{1}{1-a}\ln s - \frac{1}{1-a}\ln(n+d) + \ln A[0] + gt .$$

Equation (8) implies that $\ln k[t]$ is above *stationary lnk[t] line* for any given t. Since t always increases whatever the level of $\ln k[t]$ is, the dynamics of $\ln k[t]$ over time can be shown in Figure 1. Figure 1 shows that $\ln k[t]$ falls over time as long as the level of $\ln k[t]$ is above *stationary lnk[t] line* and rises as long as the level of $\ln k[t]$ is below *stationary lnk[t] line*.

To see the dynamics of $\ln k[t]$ over time in more detail, a simulation is undertaken by using equation (5). Solving the first-order differential equation (equation (5)) and taking logs yield:

$$(9) \qquad \ln k[t] = \frac{1}{1-a}\ln\left(k[0]^{1-a}\, e^{-(n+d)(1-a)t}(g+n+d) + sA[0]^{1-a}\,(e^{g(1-a)t} - e^{-(n+d)(1-a)t})\right) - \frac{1}{1-a}\ln(n+g+d)\ .$$

To carry out simulations, commonly used values for parameters are applied: a =0.33, g=0.02, n=0.015, d=0.03, and s=0.21. Furthermore, at this stage assume that the initial level of technology, $A[0]$, is fixed at 1. For convenience, the initial level of capital per unit of labour, $k[0]$ is set between 0.01 and 50. (Since the purpose of this analysis is to graphically capture the dynamics of $k[t]$, these choices of $k[0]$ and $A[0]$ do not cause any serious problems in the analysis.)

By substituting these particular values in equation (9), the dynamics of the level of $\ln k[t]$ can be traced. Figure 2 show the movement of $\ln k[t]$ over time for various values of $\ln k[0]$. Each curve in Figure 2 represents the movement of $\ln k[t]$ over time for the corresponding $\ln k[0]$. The higher position of the curve implies the higher level of $k[0]$.

Stationary lnk[t] line is shown in Figure 3, superimposed on the above graph. The graphical analysis in Figure 3 confirms the result shown in Figure 1. Above *stationary lnk[t] line,* $\ln k[t]$ falls over time and below *stationary lnk[t] line*, it rises. On *stationary lnk[t] line*, the growth rate of $k[t]$ is zero (i.e., *stationary lnk[t] line* intersects with the $\ln k[t]$ curves at the bottom of the $\ln k[t]$ curves).

Figure 3 also shows that $\ln k[t]$ converges to a line which is parallel to s*tationary lnk[t] line*. This line is called '*lnk*[t] line*'. Since *stationary lnk[t] line* is given by equation (6), the growth rate of $k[t]$ on *lnk*[t] line* is g. Therefore, $k[t]$ converges to a balance growth path with the growth rate of g. The level of $k[t]$ on the balanced growth path is therefore given by $k^*[t]$. The important fact here is that the position of *lnk*[t] line* depends on that of *stationary lnk[t] line* since *stationary lnk[t] line* governs the dynamics of $\ln k[t]$. The higher position of *stationary lnk[t] line* leads to the higher position of *lnk*[t] line*.

To analyze the impact of the level of technology on the dynamics of $k[t]$, $\ln k[t]$ curves can be traced with three different levels of $A[0]$, ($A[0]$ = 1, 11, and 21), with the same values of s, a, n, g, and d as before, and the level of $k[0]$ fixed at 15. Figure 4 shows the analysis. $\ln k[t]$ curves 1, 2, and 3 correspond to the levels of A[0] of 1, 11, and 21, respectively. *Stationary lnk[t] lines* 1, 2 and 3 correspond to the levels of $A[0]$ of 1, 11, and 21, respectively. As in Figure 3, each $\ln k[t]$ curve converges to its own *lnk*[t] line*. According to Figure 4, the higher level of $A[0]$ implies the higher position of *stationary lnk[t] line*. Since the position of *lnk*[t] line* depends on that of *stationary lnk[t] line;*

$\ln k[t]=(1/(1-a)) \ln s - (1/(1-a)) \ln(n+d) + \ln A[0] + g\,t)$, the balanced growth path value of $k[t]$ at any given t, $k[t]^*$, depends on a, n, d, s, g, and $A[0]$. Thus, assuming a, d, and g are the same across countries, not only s and n but also $A[0]$ affect the balanced path value and the growth rate of $k[t]$ at any given time.

Finally, I consider a long run investment policy and convergence from above the balanced growth path. Figure 5 shows that the dynamics of $\ln k[t]$. $\ln k[t]$ curve *1* shows the dynamics of $\ln k[t]$ when $\ln k[0]$ is less than $\ln k^*[0]$, and $\ln k[t]$ curve *2* shows the dynamics of $\ln k[t]$ when $\ln k[0]$ is greater than $\ln k[0]^*$. Both of these curves have the same levels of $A[0]$, s, n, and d . Thus, these two curves converge to the same *lnk*[t] line*. At point A, $t = \infty$. Points C and D show the starting points for each curve. At point B, $(\mathbf{d}\ln k[t] / \mathbf{d}t) = 0$. At around Point A, $\ln k[t] \cong \ln k^*[t]$. Notice that there are three kinds of paths. Between C and B (Path 1), $\ln k[t]$ is declining and the growth rate of $k[t]$ is increasing. Between B and A (Path 2), $\ln k[t]$ and the growth rate of $k[t]$ are both increasing. Between D and A (Path 3), $\ln k[t]$ is increasing and the growth rate of $k[t]$ is decreasing. Considering features of these three paths, we can induce some results about investment policy. If an investment does not contribute to an increase in the level of technology and a country is on Path 1, the investment policy for such a country can hardly been justified, at least in the long run, since this policy pushes the county away from her balanced growth path. Even worst, such a policy could cause a negative growth in the long run. The only case that justifies such investment policies is when a county is on Path 3. If a country is on Path 3, the investment in the county pushes her towards her balanced growth path and the country could also have a positive growth in the long run. The best policy for any economy is the one that shifts *stationary lnk[t] line* upwards (this also means shifting $\ln k^*[t]$ line up).

Cho and Graham (1996) have tested the neo-classical growth model and have found that many countries (especially, poor countries) have been converging to their balanced growth paths from above. On the basis of this finding, Aghion and Howitt (1998, Ch.1) and Temple (1998)) have pointed out the shortcomings of the neo-classical growth models. They argue that theoretical failure of the Solow model is that convergence from above the balanced growth path should not be happening in the real world although the model predicts it. They argue that countries should have been running down their capital-labour rations over time to reach their balanced growth paths from above but this is not a plausible phenomenon. However, I found this argument is not convincing. In Figure 5, Path 2 (the curve between B and A) shows the path for converging from above the balanced growth path. The economy on Path 2 would not be running down its capital-labour ratio over time but it would be increasing at lower rate than g over time. The economy experiences a positive growth rate of capital per labour for a relatively long period before reaching the balanced growth path. The important point is that those countries should have been running down not their capital-labour ratios but their capital-'*effective*' labour ratios, to reach their balanced growth path from above.

C. Graphical Analysis for the Speed of Convergence

As Figures 3 and 4 show, an economy converges to its own balanced growth path regardless of where k and A starts. Figure 6 shows the dynamics of economy in the (**d**(ln$k$*[t] - ln$k$[t])/**d**t , ln$k$*[t] - ln$k$[t]) space, corresponding to the analysis in Figure 5. $X[t]$ and $\dot{X}[t]$ denote (ln$k$[t]* - ln$k$[t]) and (**d**(ln$k$*[t] - ln$k$[t])/**d**t), respectively.

The speed of convergence is defined as how rapidly a distance between k*[t] and k[t] vanishes over time. This is a concept of conditional convergence.

Thus, the convergence coefficient can be given by:

$$(10)\qquad b[t] = -\frac{\dot{X}[t]}{X[t]}.$$

As we can see in Figure 6, if the economy starts below the balanced growth path, the speed of convergence gets slower over time, but if the economy starts above the balanced growth path, it gets faster over time. Figure 6 also shows that the absolute value of the slope of the curve can be a good approximation of $b[t]$ around the balanced growth path (i.e. around point A). Therefore, $b[t]$ around the balanced growth path can be given by:

$$(11)\qquad b[t] = -\frac{d\dot{X}[t]}{dX[t]}.$$

After some manipulation (see Appendix 1), equation (11) can be expressed as:

$$(12)\qquad b[t] = -\frac{d\dot{X}[t]}{dX[t]}$$

$$= -\frac{d\left(\frac{d(\ln k^*[t] - \ln k[t])}{dt}\right)}{d(\ln k^*[t] - \ln k[t])} = -\frac{d\left(\frac{d(\ln y^*[t] - \ln y[t])}{dt}\right)}{d(\ln y^*[t] - \ln y[t])}$$

$$= \frac{d\left(\frac{\dot{y}[t]}{y[t]}\right)}{d\left(\ln\frac{y^*[t]}{y[t]}\right)} = (1-a)(n+g+d)\left(\frac{y^*[t]}{y[t]}\right)^{\frac{1-a}{a}}.$$

Equation (12) shows that $b[t]$ is constant at the rate of (1- a) (n + g + d) when the economy is on the balanced growth path. (This is also shown by Barro & Sala-i-Martin (1995, Ch.1).) Later in this paper, equation (12) will be used to derive a specific equation to test convergence.

3. MRW'S TESTS AND THEIR SHORTCOMINGS

Many empirical tests for the Solow model are based on MRW equations. (MRW also tested the augmented Solow model with human capital).

Their equations are given by:

(13) $\ln y_i = a + (a\ /1\text{-}a)\ \ln s_i - (a\ /1\text{-}a\ )\ \ln(n_i\text{+}g\text{+}d) + e_i$,

(14) $$\left(\ln\frac{y[t]}{y[0]}\right)_i = (1-e^{-bt})a + gt + (1-e^{-bt})\frac{a}{1-a}\ln s_i$$

$$-(1-e^{-bt})\frac{a}{1-a}\ln(n+g+d)_i - (1-e^{-bt})\ln y[0]_i + (1-e^{-bt})e_i$$

where i indexes countries, y is the income per capita in 1985, e is a country specific shock, and a is a constant term. Equation (13) is their estimated equation for the test of steady state dynamics and equation (14) is for the test of conditional convergence. The term, b, in equation (14) is the convergence coefficient which is obtained by the first-order approximation around the balanced growth path, (b = (1-a) (n+g+d)). MRW found the implied b from the coefficient on $\ln y[0]$ (thus, they generalized b across countries). In setting these equations, they assume that $\ln A[0] = a + e$. That is, they try to explain the variation in the initial level of technology across countries by using the error term. However, this is not a good method to be used to test the Solow model because the difference in the initial level of technology has an important effect on the level and the growth rate of income per capita (as Section 1 shows). Thus, we should not simply rely on the error term to measure the variation in $A[0]$. Also, since the OLS regression is based upon minimizing the residuals sum of squares, if we apply the OLS regression method in their specification it implies forcing the cross-country difference in the initial level of technology to be minimized. This implies ignoring the importance in the difference in the initial level of technology. Thus, their method is appropriate to test the model only if we assume that there are no technological differences across countries (i.e. technologies are nonrival and non-excludable goods). However, since it is not reasonable to assume no systematic technological differences across countries, their estimates for coefficients would not be reliable.

Another important criticism is that MRW assume that a country's initial level of technology is not correlated with the regressors in their estimated equation. That is, the error term is uncorrelated with independent variables. This is a very weak assumption particularly in equation (14). Almost certainly, there is positive correlation between e and $\ln y[0]$. Thus, the estimated coefficients may be seriously biased. (MRW mentioned about this problem in their article.) Aghion and Howitt (1998, Ch.1) argue that since this assumption seems not to be practical in a real world, this unobservable initial level of technology should be omitted from the regressions in order to obtain the unbiased coefficient estimates.

In the following sections of this paper, therefore, I will attempt to estimate the Solow model by taking a different approach from MRW. Firstly, it is assumed that technology is not a worldwide good but rather a domestic good and that technological accumulation is governed only by time. This means all countries face the common growth rate of technological progress, but not the same initial level of technology. Also assume that there is no technological diffusion across countries. This implies that technologies are completely excludable across countries. One possible explanation of such excludability is that the followers may face a large adaptation cost when they try to achieve the leading level of technology. Thus, the differences in the level of technology persist over time. Based upon these assumptions, I will try to derive the estimated equation in the way that the unobservable initial level of technology is omitted from regression. In other words, the impact of difference in the initial level of technology is incorporated in the estimated equation without relying on the error term.

4. EMPIRICAL TESTS FOR STEADY STATE DYNAMICS

A. The Specification

From equations (1), (2), (3), and (4), the equation for the motion of (*K*/*AL*) can be expressed as:

(15) $$\dot{\hat{k}}[t] = s\hat{k}[t]^{a} - (n+g+d)\hat{k}[t],$$

where $\hat{k}[t]$ is (*K*/*AL*), capital per effective labour. The standard approach to the Solow model shows that $\dot{\hat{k}}[t]$ is equal to zero at the balanced growth path. Thus equation (15) can be rewritten as:

(16) $$k^{*}[t] = s^{\frac{1}{1-a}} A[t](n+g+d)^{\frac{-1}{1-a}},$$

where $k^{*}[t]$ denotes the balanced growth path level of capital per labour, *K*/*L* at time *t*. Substituting equation (3) into this expression and taking logs yield:

(17) $$\ln k^{*}[t] = \left(\frac{1}{1-a}\right)\ln s - \left(\frac{1}{1-a}\right)\ln(n+g+d) + \ln A[0] + gt.$$

From equation (1), the intensive form of the production function is given by:

(18) $$y[t] = A[0]^{1-a} e^{(1-a)gt} k[t]^{a}.$$

Thus, the balanced growth path level of $y[t]$ is given by:

(19) $$y^{*}[t] = A[0]^{1-a} e^{(1-a)gt} k^{*}[t]^{a}.$$

Taking the anti-log of equation (17) and substituting it into equation (19) yields:

$$(20) \qquad y^*[t] = A[0] s^{\frac{a}{1-a}} (n+g+d)^{-\frac{a}{1-a}} e^{gt} .$$

The initial level of output per capita is given by (from equation (18)):

$$y[0] = A[0]^{1-a} k[0]^{a} .$$

Solving this expression with respect to $A[0]$ yields:

$$(21) \qquad A[0] = \left(\frac{k[0]}{y[0]} \right)^{-\frac{a}{1-a}} y[0] .$$

Substituting equation (21) into equation (20) and taking logs gives:

$$(22) \qquad \left(\ln \frac{y^*[t]}{y[0]} - gt \right)_i = -\frac{a}{1-a} \left(\ln \frac{k[0]}{y[0]} \right)_i + \frac{a}{1-a} \ln s_i - \frac{a}{1-a} \ln(n+g+d)_i + e_i ,$$

where i indexes countries. Equation (22) is the specification used in this section. Note that the error term, e_i, does not represent the variation in the initial level of technology. Assuming at year time, t, all countries are at their balanced growth paths, t represents the time length between starting year and year, t. All countries face the same rates of technological progress and depreciation.

It is assumed that the rate of saving, the population growth rate, and the initial level of capital-output ratio are independent of the error term (e_i is a white noise) and that there is no serious multicollinearity among independent variables. Notice that there is no constant term in equation (22). If (gt) is moved to the right hand side of equation (22) and treated as a constant term, the coefficients estimates would be seriously biased because gt and $\ln(n+g+d)$ are dependent each other. Equation (22) is, therefore, estimated with ordinary least squares by forcing it not to have a constant term. The restricted version of the specification is given by:

$$(23) \qquad \left(\ln \frac{y^*[\mathrm{t}]}{y[0]} - gt \right)_i = -\frac{a}{1-a} \left(\ln \frac{k[0]}{y[0]} - \ln s + \ln(n+g+d) \right)_i + e_i .$$

The interpretation of equation (22) is straightforward. The gap between the initial level of income per labour and the balanced growth path level of income per labour at time t is positively associated with the saving rate and negatively with the population growth rate. The important point here is that the initial level of capital-output ratio is negatively associated with $\ln(y^*[t]/y[0])$. What does this imply?

Rewriting equation (21) yields:

$$(24)\qquad \frac{A[0]}{y[0]} = \left(\frac{k[0]}{y[0]}\right)^{\frac{-a}{1-a}} .$$

The term $A[0]/y[0]$ measures the technological level relative to the income per capita at the initial time. Equation (24) implies that the lower value of $k[0]/y[0]$ corresponds to the higher value of $A[0]/y[0]$. Intuitively, if Country A has a lower capital-output ratio than Country B, Country A is producing goods more effectively than Country B. Thus, assuming all countries are on the balanced growth path at time t, the country with the lower initial capital-output ratio would have a larger increase in the income per labour between the period 0 and t due to the greater effectiveness in its production process.

B. Data

Data used in this paper are from the Summers and Heston data set version 5.6 (which is described in Summers and Heston (1991)), the Barro-Lee data set (used in Barro and Lee (1994)), and the King-Levine data set (used in King and Levine (1994)). The average growth rate of the working-age population is used for the population growth rate, n, where working age is defined as 15 to 64. These data are constructed by using the Barro-Lee data set. The data for the saving rate, s, (the average share of real investment in real GDP over the period of 1960-1985) and GDP per the equivalent adult, y, (in 1960 and 1985) are from the Summers and Heston data set. The data for capital-output ratio, k/y, (in 1960 and 1985) are from the King-Levine data set.

The sample covers MRW's 'Non-oil' countries in which the dominant industries are not oil production. It consists of 95 'Non-oil' countries for which all necessary data are available. The data set covers the period between 1960 and 1985. (see Appendix 2)

C. Results

First of all, the sample is divided into three sub-sample groups. Using equation (23) and substituting 0 for t yields:

$$(25)\qquad \left(\frac{1-a}{a}\right)\left(\ln\frac{y^*[0]}{y[0]}\right)_i = -\left(\ln\frac{k[0]}{y[0]} - \ln s + \ln(n+g+d)\right)_i + e_i .$$

Equation (25) shows whether a country was initially below or above its own balanced growth path. Assuming $0<a<1$ and treating year 1960 as the initial year, the countries with negative value of $(\ln(k[0]/y[0]) - \ln s + \ln(n+g+d))$ were below their own balanced growth path in 1960 and those with positive value of $(\ln(k[0]/y[0]) - \ln s + \ln(n+g+d))$ were above their own balanced growth path in 1960. It turns out that all OECD counties and 41 other countries were below their balanced growth paths in 1960 and the remaining 32 countries were above their own balanced growth path in 1960. Therefore, the sample is divided into three sub-sample groups; ‘OECD’ (22 countries), ‘Below’ (41 countries), and ‘Above’ (32 countries).

I estimate equations (22) and (23), assuming the technological growth rate, g, is 0.02, the depreciation rate, d, is 0.05, and all countries were on their own balanced growth path in year 1985.[3] Table 1 shows the results.

The results in Table 1 support the Solow model for the ‘OECD’ and the ‘Above’ samples. All the coefficients in the ‘OECD’ and the ‘Above’ samples are highly significant and have the signs predicted by the model. The F tests do not reject the restrictions at 5% significance level. The raw R^2 for the restricted regression is 0.608 for the ‘OECD’ sample and 0.623 for the ‘Above’ sample.[4] Most importantly, the implied a, the capital share, is 0.31 for the ‘OECD’ sample and 0.351 for the ‘Above’ sample. Thus, without introducing human capital into the model, the data strongly support the prediction that a is 1/3.[5]

Contrary to the ‘OECD’ and the ‘Above’ samples, the data for the ‘Below’ sample fails to support the model. The coefficient for the restricted regression is not significant and the raw R^2 is 0.009. The implied a is far below the prediction. Also, the F test rejects the restriction at 1% significance level.

5. EMPIRICAL TESTS FOR CONVERGENCE

A. The Specification

From equation (12), the convergence coefficient is given by:

$$(26) \qquad b[t] \cong (1-a)(n+g+d)\left(\frac{y[t]}{y^*[t]}\right)^{\frac{a-1}{a}} .$$

Taking logs of equation (26) gives:

$$(27) \qquad \ln b[t] = \ln(1-a) + \ln(n+g+d) - \frac{1-a}{a}(\ln y[t] - \ln y^*[t]) .$$

From equation (22), we can obtain:

$$(28) \qquad \ln y^*[t] = \ln y[0] - \frac{a}{1-a}\ln\frac{k[0]}{y[0]} + \frac{a}{1-a}\ln s - \frac{a}{1-a}\ln(n+g+d) + gt .$$

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[3] I assume that δ is 0.05 as Barro and Sala-i-Martin (1992a, 1992b). They used the value reported by Jorgenson and Yun (1986, 1990). MRW (1992) take δ as 0.03, but this difference does not cause any serious problem in the regression analysis in this paper.

[4] The values of raw R^2 are calculated according to the definition: $Raw \ \ R^2 = 1-\left(\sum_i e_i / \sum_i Y_i\right)$, where e is the residual from the regression without a intercept and Y is the dependent variable observed.

[5] One of the main reasons that MRW (1992) rejected the strict Solow model is that the estimated α for the Solow model without introducing human capital is much too high to be consistent with the conventional value of capital share.

Substituting this expression into equation (27) and arranging it yields:

$$(29)\qquad \ln\frac{y[t]}{y[0]}=\frac{a}{1-a}\ln(1-a)-\frac{a}{1-a}\ln b[t]+gt-\frac{a}{1-a}\ln\frac{k[0]}{y[0]}+\frac{a}{1-a}\ln s\ .$$

Since $b[t]$ is not observable, it is assumed that $b[t]$ is the generalized value across countries (i.e. the b value at time t is the same across countries). Therefore, the estimated equation is given by:

$$(30)\qquad \left(\ln\frac{y[t]}{y[0]}-gt\right)_i=C-\frac{a}{1-a}\left(\ln\frac{k[0]}{y[0]}\right)_i+\frac{a}{1-a}\left(\ln s\right)_i+e_i\ ,$$

where C is a constant term, (C = (a /1-a) (ln (1-a) – ln $b[t]$)). The interpretation of this equation is similar to that of equation (22) in Section 3. If Country A has a lower capital-output ratio than Country B at the starting point, Country A can produce goods more effectively than Country B. Thus, Country A would increase the income per labour between the period 0 and t more than Country B, due to the greater effectiveness in its production process.

The restricted version of the equation is given by:

$$(31)\qquad \left(\ln\frac{y[t]}{y[0]}-gt\right)_i=C-\frac{a}{1-a}\left(\ln\frac{k[0]}{y[0]}-\ln s\right)_i+e_i\ .$$

By estimating equation (31), using OLS, an estimate for (a /1-a) can be obtained. This, in turn, can give the estimate for b by looking at the estimate for the constant term.

This estimate for b can be a better estimate than the conventional estimates for b which use the first order approximation around balanced growth path. The reason is that the method taken here allows the economy to be further away from the balanced growth path to measure b.

C. Results

Before estimating equations of (30) and (31), a supplementary test of convergence is introduced. By substituting 0 for t in equation (27) and using the obtained expression with equation (25), we can obtain:

$$(32)\qquad \ln b[0]_i=\ln(1-a)-\ln\left(\frac{k(0)}{y(0)}\right)_i+\ln s_i\ .$$

Substituting the capital-output ratio at t = 0 and the average saving rate for each country into equation (32) and assuming a is 1/3, we can obtain $\ln b[0]$ for each country. Thus, the generalized b across countries at t = 0 can be obtained by simply taking the mean value across countries. Taking year 1985 as the initial year, Table2 gives the results.

Table 2 reports that the generalized b value in year 1985 is 0.067 for the 'OECD' sample, 0.048 for the 'Above' sample, and 0.063 for the 'Below' sample. Thus, if the model describes the mechanism of economic growth well, the OLS regression estimates on equations (30) and (31) should give the estimated b values which are close to these mean b values and also give the estimated a values which are close to 1/3.

Empirical results of regressions to test convergence are given in Table 3. The results in Table 3 show that the data for the 'OECD' and 'Above' samples seem to support the Solow model. All coefficients for those samples are highly significant and have the signs predicted by the model. Although the R^2 is relatively low for the 'OECD' sample, the restrictions for those samples are not rejected at the 1% significance level (for the 'Above' sample, the restriction is not rejected at 5% significance level) and the implied bs are reasonably close to the values reported in Table 2. In the case of the 'Below' sample, the results in Table 3 do not support the model. Although all coefficients have the signs predicted by the model, the coefficient on ln(k/y60) in the unrestricted regression and all coefficients in the restricted regression are not significant at the 5% significance level. The R^2, 0.051, in the restricted regression is poor and the F test rejects the restriction. The implied b is far away from the value given by the results in Table 2.

The implied as for 'OECD' sample and 'Above' sample are very close to the conventional value of capital share, 0.33. The Solow model's prediction about the speed of convergence around the balanced growth path is $b = (1-a)(n + g + d)$. The estimated b for 'OECD' and 'Above' samples is roughly 0.06. Suppose that the parameters take on the baseline values: n = 0.01, g = 0.02, and d = 0.05, therefore, to match the Solow model's prediction a is required to be about 0.35. The results are close to the conventional value of capital share. Thus, the estimated b and a do not contradict with the model's prediction about the speed of convergence.

6. INTERPRETATION FOR THE OBTAINED RESULTS

Assuming *complete cross-country excludability of technologies*, the results from Sections 3 and 4 show that the Solow model explains the growth mechanism for the 'OECD' and 'Above' samples quite well but not for the 'Below' sample. This section attempts to find a possible explanation for these results. I consider particularly a diffusion of technology from a leading country to followers.

First, the distance from the balanced growth path in 1960 and 1985 for the 'OECD' sample and the 'Below' sample is considered. Figures 7 and 8 describe the findings.

Figure 7 plots a fraction of a distance from the balanced growth path for each 'OECD' sample country both in 1960 and 1985 and Figure 8 shows that for each 'Below' sample country in 1960 and 1985. The fraction of distance from the balanced growth path is measured by $(\ln y - \ln y^*)\ ((1-a)/a)$, assuming a is the same across countries and constant over time. (Here, the fraction of distance from the balanced growth path is calculated on the basis of equation (23) by treating both year 1960 and year 1985 as the initial time.) The country is on the balanced growth path if it is on the horizontal line at 0. The further the county is down from the line, the larger the distance from the balanced growth path is. Comparing Figure 7 with Figure 8, it is found that more than a half of countries in the 'Below' sample overshoot the balanced growth path in 1985 although no country in the 'OECD' sample does. There are several possible reasons for this overshooting. The first is that the saving rates in those countries increase over time. The second is that the population growth rates or the depreciation rate in those countries decrease over time. The third is that the levels of technology in those countries grow more than 2 per cent per year (the growth rate of technology may not be constant). The fourth is any mixture of these three. In the following parts of this section, the third idea is considered.

A similar approach taken in Section 1 is adopted to consider this possibility. Equation (16) shows that $\ln k^*[t]$ line is given by:

$$\ln k^*[t] = \left(\frac{1}{1-a}\right)\ln s - \left(\frac{1}{1-a}\right)\ln(n+g+d) + \ln \mathrm{A}[0] + gt \ .$$

Assuming a, s, n, g, and d are constant over time, *lnk*[t] line* is a straight line in the ($\ln k[t]$, t) space. Assume $\ln A$ jumps up three times at time t_1, t_2, and t_3 in a finite time horizon and the degree of the increase gets smaller each time although A grows at a constant rate, g, for the rest of time. These jumps shift *lnk*[t] line* up each time. The analysis of the dynamics of $\ln k[t]$ is shown in Figure 9.

In Figure 9, the economy initially starts at point A. Between 0 and t_1, the economy's balanced growth path level of capital-labour ratio is shown by $\ln k^*[t]$ line$_0$. Thus, $\ln k[t]$ converges towards *lnk*[t] line* over time if there is the constant growth rate of technology. The dynamics of $\ln k[t]$ between 0 and t_1 is given by the curve A-B. Assume that the level of technology suddenly jumps up at t_1 and $\ln k^*[t]$ line$_0$ shifts up to $\ln k^*[t]$ line$_1$. $\ln k[t]$ now converges towards $\ln k^*[t]$ line$_1$ and the dynamics is shown by the curve B-C. At t_2, $\ln A[t]$ jumps up again but this time the jump is smaller than at t_1. The same mechanism is applied. The dynamics of $\ln k[t]$ is now shown by the curve C-D. At time T, the economy reaches at point E. If there is no jump at all, the economy follows $\ln k[t]$ curve$_0$ and reaches at point G at time T. Therefore, the economy overshoots the original balanced growth path level at time T, if there are sudden increases in the level of technology. The vertical distance between E and F shows the degree of the *overshoot*. If we assume that there are infinitely many jumps and the interval of jump is infinitely short, we can obtain the smooth path of $\ln k[t]$ over time. It also means that the technological level continuously increases and it shows decreasing returns to time, t.

The above idea is used to analyze the dynamics of technological progress. The assumption of *partial cross-country excludability of technologies* is now introduced. Firstly, it is assumed that there is one technologically leading country and the others are followers. As before technologies are assumed not to be worldwide goods but rather domestic goods. However, this time, assume that not all countries have the common and constant growth rate of technological progress. Although technological accumulation is governed only by time (i.e. *initially* all countries face a common and constant growth rate of technology, g, over time), the technical diffusion from the leader benefits some countries. Some countries can easily absorb the cutting-edge technology and take a full use of it. The adoption of the leading technology can push the country's technological level up over time. A larger gap in the technological level between the leader and the follower can induce a greater increase in the follower's level over time. This seems reasonable if we think about the technological imitation. If the follower has a similar level of technology as the leader, there is not much to be adapted or imitated by the follower. Another point is that it is very hard to adapt or imitate the leading technology if the countries do not have baseline technology since those countries would face a large adaptation cost. Thus, there would be no technological diffusion to those countries and their technological accumulation is governed only by time. Figure 10 summarizes this idea.

The top thick straight line in Figure 10 represents leader's $\ln A[t]$ line. The thick line in the lower part of Figure 10 represents threshold $\ln A[t]$ line. There is no technological diffusion below this line. In the range between these two lines, the technological diffusion takes place. The function for technology in this area satisfies:

$$(33) \qquad \frac{d}{dt}\ln A[t] > g, \frac{d^2}{dt^2}\ln A[t] < 0, \lim_{t\to 0}\frac{d}{dt}\ln A[t] = \infty, and \lim_{t\to\infty}\frac{d}{dt}\ln A[t] = g,$$

where g is the common and constant growth rate of technology.

Considering above ideas, it is now possible to explain why the data for the 'Below' sample do not support the Solow model. If the level of technology in the country starts somewhere further below leader's $\ln A[t]$ line and above threshold line, a large degree of technological diffusion would take place over time and the economy would largely overshoot the balanced growth path with starting from the below balanced growth path. If the technological level in the country is just below leader's $\ln A[t]$ line, there would be only small degree of diffusion and the country would not overshoot. If the technological level in the country starts below threshold $\ln A[t]$ line, the country would exactly follow the Solow model.

It is reasonable to assume that the 'OECD' sample countries' technological levels start just below leader's $\ln A[t]$ line. Thus, the growth rates of technology in these countries are fairly constant over time and similar across countries. This could be the reason why the 'OECD' sample supports the model well. As to other countries, it is found that the mean values of the average growth rates of population and the average investment rates are very similar for both the 'Below' sample and the 'Above' sample. Therefore considering equation (16):

$$\ln k^*[t] = \left(\frac{1}{1-a}\right)\ln s - \left(\frac{1}{1-a}\right)\ln(n+g+d) + \ln \mathrm{A}[0] + gt ,$$

the 'Above' sample countries are expected to have much smaller $\ln A[0]$ to be above their balanced growth paths in the initial year than the 'Below' sample countries. Therefore, the 'Above' sample countries' technological levels would start below threshold $\ln A[t]$ line and the 'Below' countries' levels would start between leader's $\ln A[t]$ line and threshold $\ln A[t]$ line in Figure 10. Therefore, the 'Below' sample countries would benefit from the technological diffusion and the growth rates of technology are not constant over time and differ across those countries. This could be the reason why the 'Below' sample data show the poor results. Therefore, it can be said that there are two kinds of reasons for convergence for middle income countries: diminishing returns to capital and technological diffusion. Barro and Sala-i-Martin (1997) formally model this idea although they do not consider the point that some countries may not benefit from technological diffusion from a technological leader. Above all, the Solow model could be improved if the idea of technological diffusion is introduced into the model's framework. Also, as Quah (1993a, 1993b) argues, this could be the reason that the world is heading toward a bi-modal income distribution: 'convergence clubs of rich and poor'.

7. SUMMARY AND CONCLUSION

This paper analyses the Solow model both empirically and theoretically by taking a new approach. Several important implications have been made.

First, the paper shows the importance of a variation in the initial level of technology in the Solow model. Assuming that all countries face constant and common growth rate of $A[t]$, the balanced growth path and the growth rate of income per capita are sensitive to the initial level of technology.

Second, theoretical implication of convergence above balanced growth path does not necessarily contradict real world events. Recently, Temple (1998) and Aghion and Howitt (1998, Ch.1) have argued that the Solow model is not appealing because the model's prediction about convergence from above can only be achieved by running down capital-labour ratio on assuming constant rates of investment and population growth. Their point is that running down capital stock is not plausible in the real world. However, this paper claims that convergence from above does not necessarily need constantly reducing capital-labour ratio to reach the balanced growth path. Countries converging from above could face a positive growth rate of capital per labour for a relatively long period before reaching their balanced growth paths.

Third, a new method to test the Solow model is constructed by paying a great attention to cross-country differences in initial levels of technology. Assuming *complete cross-country excludability of technologies*, the results show that the Solow model without human capital augmentation explains growth mechanisms well for the 'OECD' sample and the 'Above' sample. That is, convergence due to diminishing returns to capital is observed in these two samples. The implied b is about 0.06 which is much higher than conventional estimates of b, 0.02.

Finally, the poor results for the 'Below' sample can be explained if we assume those countries largely benefited from technological diffusion - *partial cross-country excludability of technologies* -. In other words, growth rates of technology are far from constant in those countries. As for the 'Below' sample countries, thus, technological diffusion should be considered in order to analyze convergence. Therefore, although the Solow model seems to fit the data well for some countries, there are still grounds for improvement. This paper has shown that introducing technological diffusion into the model improves our understanding of the mechanism of growth.

<u>Appendix 1</u>

The coefficient of the speed of convergence is given by:

$$\beta[t] \cong -\frac{d\dot{X}[t]}{dX[t]} = -\frac{d\left(\frac{d(\ln k^*[t]-\ln k[t])}{dt}\right)}{d(\ln k^*[t]-\ln k[t])}. \qquad \text{---(a)}$$

Using the fact:

$$\ln y[t] = (1-\alpha)\ln A[t] + \alpha \ln k[t]$$
$$\ln y^*[t] = (1-\alpha)\ln A[t] + \alpha \ln k^*[t],$$

the distance between $\ln k^*[t]$ and $\ln k[t]$ is given by:

$$\ln k^*[t] - \ln k[t] = \frac{1}{\alpha}(\ln y^*[t] - \ln y[t]). \qquad \text{---(b)}$$

Substituting equation (b) into equation (a) yields:

$$\beta[t] = -\frac{d\left(\frac{d(\ln y^*[t]-\ln y[t])}{dt}\right)}{d(\ln y^*[t]-\ln y[t])} = -\frac{d\left(g-\frac{\dot{y}[t]}{y[t]}\right)}{d\ln\frac{y^*[t]}{y[t]}} = \frac{d\,\frac{\dot{y}[t]}{y[t]}}{d\ln\frac{y^*[t]}{y[t]}}. \qquad \text{---(c)}$$

Since $y[t] = A[t]^{1-\alpha} k[t]^{\alpha}$, the growth rate of output per labour is given by:

$$\frac{\dot{y}[t]}{y[t]} = (1-\alpha)g + \alpha\frac{\dot{k}[t]}{k[t]}. \qquad \text{---(d)}$$

By using the equation for the capital accumulation, the growth rate of capital per labour is given by:

$$\frac{\dot{k}[t]}{k[t]} = sA[t]^{1-\alpha} k[t]^{\alpha-1} - (n+\delta). \qquad \text{---(e)}$$

Substituting equation (e) into equation (d) yields:

$$\frac{\dot{y}[t]}{y[t]} = (1-a)g + a(sA[t]^{1-a}\,k[t]^{a-1} - (n+d)). \qquad \text{---(f)}$$

Since on the balanced growth path $sA[t]^{-a}\,k[t]^{a} - (n+g+d)k[t]A[\mathrm{t}]^{-1} = 0$,

$$k*[t]^{a-1} = (n+g+d)s^{-1}A[t]^{a-1}.$$

Solving this equation for $A[t]^{a-1}$ yields:

$$A[t]^{1-a} = (n+g+d)s^{-1}k*[t]^{-(a-1)}. \qquad \text{---(g)}$$

Substituting equation (g) into equation (f) yields:

$$\frac{\dot{y}[t]}{y[t]} = (1-a)g + a((n+g+d)\left(\frac{k[t]}{k*[t]}\right)^{a-1} - (n+d)). \qquad \text{---(h)}$$

Rewriting equation (b) yields:

$$\frac{k[t]}{k*[t]} = \left(\frac{y[t]}{y*[t]}\right)^{\frac{1}{a}}.$$

Substituting this expression into equation (h) yields:

$$\frac{\dot{y}[t]}{y[t]} = (1-a)g + a(n+g+d)\left(\frac{y*[t]}{y[t]}\right)^{\frac{1-a}{a}} - a(n+d). \qquad \text{---(i)}$$

According to equation (c), the coefficient for the speed of convergence is now given by:

$$\begin{aligned} b[t] &= \frac{d}{d\ln\frac{y*[t]}{y[t]}}\left((1-a)g + a(n+g+d)\left(\frac{y*[t]}{y[t]}\right)^{\frac{1-a}{a}} - a(n+d)\right) \\ &= \left(\frac{y*[t]}{y[t]}\right)\left(\frac{1-a}{a}a(n+g+d)\left(\frac{y*[t]}{y[t]}\right)^{\frac{1-2a}{a}}\right) \\ &= (1\text{-}a)(n+g+d)\left(\frac{y[t]}{y*[t]}\right)^{\frac{a-1}{a}}. \end{aligned}$$

When $y[t] = y*[t]$ on the balanced growth path, $b[t]$ is constant at $(1\text{-}a)(n+g+d)$.

Appendix 2

S-H code	Country	Sample	OECD	Above	Below	ln(GDP60)	ln(GDP85)	n	ln(s)	ln(k/y60)	ln(k/y85)
1	Algeria	1	0	0	1	7.699389	8.260493	0.0288436	-1.522731	0.3370021	0.951568
2	Angola	1	0	1	0	7.071573	6.818924	0.0230568	-3.313609	-0.004866	0.511218
3	Benin	1	0	0	1	7.230563	7.276556	0.0231055	-2.734552	-1.112163	-0.41742
4	Botswana	1	0	0	1	6.553934	8.037867	0.033825	-1.621238	0.1484519	0.499608
5	Burkina	0	0	0	0	6.356108	6.452049		-2.67644		
6	Burundi	1	0	0	1	6.687109	6.520621	0.0163825	-3.122692	-1.011027	-0.13958
7	Cameroon	1	0	1	0	6.689599	7.549083	0.0250637	-2.554009	-0.178757	-0.0589
8	Cape ver	0	0	0	0	6.395262	7.187657	0.0231173	-1.443761		
9	Central African Rep.	1	0	1	0	6.77079	6.683361	0.0201385	-2.687682	-0.035596	-0.04865
10	Chad	1	0	1	0	6.849066	6.249975	0.0187643	-3.870586	0.5483479	0.584328
11	Comoros	0	0	0	0	6.54535	6.728629		-1.950845		
12	Congo	1	0	1	0	7.252762	8.14555	0.0266368	-2.169871	0.3537015	-0.01577
13	Egypt	1	0	1	0	6.934397	7.798112	0.0266048	-3.086668	-0.713501	-0.67321
14	Ethiopia	1	0	0	1	5.802118	5.955837	0.0238531	-3.005006	-1.204393	-0.74748
15	Gabon	0	0	0	0	7.666222	8.50208	0.030138	-1.48671	1.077785	1.25964
16	Gambia	0	0	0	0	6.621406	6.883462	0.0302181	-3.132345	-2.34683	-0.4588
17	Ghana	1	0	1	0	7.047517	6.940222	0.0254065	-2.757927	-0.079105	-0.18526
18	Guinea	0	0	0	0	6.556778	6.811244	0.0181776	-2.771974		
19	Guinea-B	0	0	0	0	6.428105	6.699501		-1.71994		0.846187
20	Cote d'I	1	0	0	1	7.249215	7.601402	0.0381005	-2.114511	-0.293597	0.085891
21	Kenya	1	0	1	0	6.775366	6.981006	0.0366445	-1.813062	0.8838167	0.33737
22	Lesotho	0	0	0	0	5.97381	7.119636	0.0229097	-2.366508	-0.496496	0.02132
23	Liberia	1	0	1	0	6.813445	7.015712	0.0282933	-2.031384	1.562098	1.113921
24	Madagasc	1	0	1	0	7.312553	6.895683	0.0253705	-4.316527	-0.152826	0.109513
25	Malawi	1	0	0	1	6.200509	6.510258	0.0313452	-2.250139	-0.240785	0.069036
26	Mali	1	0	0	1	6.526495	6.54103	0.0226589	-2.842021	-0.623205	-0.3709
27	Mauritan	1	0	0	1	6.904751	6.977282	0.0218568	-1.895327	-0.515384	0.651792
28	Mauritiu	1	0	1	0	8.223627	8.521584	0.0284484	-2.267441	0.0869288	0.134077
29	Morocco	1	0	0	1	6.956545	7.807917	0.0295786	-2.405385	-0.144078	-0.13874
30	Mozambiq	1	0	1	0	7.277938	6.862758	0.0237252	-3.989985	0.1740499	0.718287
31	Niger	1	0	0	1	6.511745	6.593045	0.0292935	-2.396051	-0.152637	0.068579
32	Nigeria	1	0	0	1	6.598509	7.244227	0.0308684	-2.011064	-0.669812	0.506073
33	Rwanda	1	0	0	1	6.53814	6.930495	0.0313865	-3.417092	-1.264916	-0.76906
34	Senegal	1	0	1	0	7.194437	7.309212	0.0284739	-2.928074	-0.428068	-0.27143
35	Seychell	0	0	0	0	7.448916	8.259717		-1.834025		
36	Sierra L	0	0	0	0		7.040536	0.019734		-1.374184	-1.28396
37	Somalia	1	0	0	1	7.237778	6.734591	0.0312744	-2.467823	-0.591232	0.266303
38	South af	1	0	1	0	7.915713	8.337828	0.0243513	-1.647459	0.7449784	1.032508
39	Sudan	0	0	0	0		6.928538	0.0280652			
40	Swazilan	0	0	0	0	7.372118	7.955425	0.0281382	-2.089647	0.5783786	0.957071
41	Tanzania	1	0	1	0	6.023448	6.437752	0.0334137	-2.243228	0.4527913	0.509668
42	Togo	1	0	0	1	6.146329	6.710523	0.0283308	-1.816367	0.079191	0.695115
43	Tunisia	1	0	1	0	7.248504	8.140024	0.0268611	-1.857406	0.4904225	0.232066
44	Uganda	1	0	1	0	6.656726	6.566672	0.0369209	-3.663062	-0.820476	-1.51634
45	Zaire	1	0	1	0	6.440947	6.349139	0.0286192	-3.240257	-0.293303	0.324384
46	Zambia	1	0	1	0	7.127694	6.964136	0.0329501	-1.415486	1.920132	1.293919

47	Zimbabwe	1	0	1	0	7.155396	7.375882	0.0318062	-1.707984	1.091217	0.369689
48	Bahamas,	0	0	0	0		9.572341				
S-H code	Country	Sample	OECD	Above	Below	ln(GDP60)	ln(GDP85)	n	ln(s)	ln(k/y60)	ln(k/y85)
49	Barbados	0	0	0	0	8.099554	8.867568	0.0078857	-2.07729	0.4936444	0.76361
50	Canada	1	1	0	0	9.073375	9.768011	0.0207491	-1.456222	0.7800774	0.823349
51	Costa Ri	1	0	0	1	7.918629	8.268476	0.0416528	-1.854945	-0.089899	0.355817
52	Dominica	0	0	0	0		8.092851				
53	Dominican Rep.	1	0	0	1	7.353082	7.882315	0.0346037	-1.935808	-0.006165	0.52549
54	El Salva	1	0	0	1	7.518607	7.76472	0.0261919	-2.470549	-0.546045	-0.14233
55	Grenada	0	0	0	0		7.742402				
56	Guatemal	1	0	0	1	7.67601	7.90581	0.0297376	-2.36446	-0.235196	-0.08765
57	Haiti	1	0	0	1	7.057037	7.060476	0.0167458	-3.002679	-1.205027	-0.06736
58	Honduras	1	0	0	1	7.204893	7.501082	0.0334985	-1.954912	0.1508473	0.272585
59	Jamaica	1	0	0	1	7.713338	7.905073	0.016675	-1.471015	0.9660739	0.888657
60	Mexico	1	0	0	1	8.209309	8.871224	0.0345024	-1.772636	0.3480655	0.684784
61	Nicaragu	1	0	1	0	7.655391	7.756196	0.0350152	-2.165838	0.3375673	0.90748
62	Panama	1	0	0	1	7.607878	8.367997	0.0319234	-1.518684	0.4710518	0.749432
63	St.Lucia	0	0	0	0		7.942007				
64	St.Vince	0	0	0	0		7.996654				
65	Trinidad	1	0	1	0	8.87794	9.35988	0.0203577	-2.07239	0.4687234	0.980898
66	United State	1	1	0	0	9.368455	9.831185	0.0159835	-1.535687	0.4764255	0.581676
67	Argentin	1	0	0	1	8.570354	8.748305	0.0138014	-1.766542	-0.006398	0.368199
68	Bolivia	1	0	0	1	7.286876	7.716906	0.0256184	-1.672746	0.5529295	0.592821
69	Brazil	1	0	1	0	7.732369	8.499436	0.0313067	-1.615224	0.7385039	0.601165
70	Chile	1	0	0	1	8.186743	8.315077	0.0255105	-1.677058	0.2835649	0.365628
71	Colombia	1	0	1	0	7.692113	8.201385	0.0339916	-1.82659	0.7004856	0.484944
72	Ecuador	1	0	0	1	7.54009	8.211483	0.0351993	-1.472858	0.7576752	0.86152
73	Guyana	0	0	0	0	7.652546	7.346655	0.0226231	-1.394947	1.131886	1.469755
74	Paraguay	1	0	0	1	7.331715	7.869784	0.0353251	-2.075143	-0.543502	0.172257
75	Peru	1	0	0	1	7.853993	8.075583	0.0309596	-1.724676	0.5160865	0.615037
76	Suriname	0	0	0	0	7.86442	8.335192	0.0200714	-1.63594		0.486192
77	Uruguay	1	0	1	0	8.435766	8.430763	0.0057317	-2.022916	0.6384402	0.921551
78	Venezuel	1	0	1	0	9.01627	8.956351	0.0414427	-1.682009	0.5905827	0.65013
79	Afghanis	0	0	0	0			0.0129396		-0.559012	-0.25089
80	Bahrain	0	0	0	0		9.347665				
81	Banglade	1	0	1	0	7.087574	7.36201	0.0252912	-3.099385	-0.428771	-0.55083
82	Myanmar	1	0	1	0	5.988961	6.602588	0.0255939	-2.427364	0.1470501	0.089782
83	China	0	0	0	0	6.556778	7.301822	0.0265502	-1.629248		
84	Hong Kon	1	0	0	1	7.946264	9.394577	0.0337946	-1.596446	0.6538412	0.456308
85	India	1	0	0	1	6.862758	7.160846	0.0250537	-1.992558	0.1090391	0.535149
86	Indonesi	0	0	0	0	6.682108	7.624131		-1.921255	-1.069445	0.672985
87	Iran, I.	0	0	0	0	8.256607	8.545003	0.036938	-1.902777	0.3056521	0.592504
88	Iraq	0	0	0	0	8.401558	8.621914	0.0357552	-2.251234	0.7325891	1.444123
89	Israel	1	0	0	1	8.352319	9.197559	0.0302991	-1.289028	0.8669912	0.771019
90	Japan	1	1	0	0	8.1545	9.488654	0.0128214	-1.080848	0.347504	1.034886
91	Jordan	1	0	0	1	7.309212	8.453827	0.0278069	-1.957905	-0.94302	0.635048
92	Korea	1	0	0	1	7.041412	8.516793	0.0280082	-1.539804	-0.031858	0.795323
93	Kuwait	0	0	0	0		9.704854	0.084827		-0.767517	0.88335
94	Malaysia	1	0	0	1	7.512618	8.539346	0.033816	-1.499205	0.426683	1.019204
95	Nepal	1	0	0	1	6.654152	7.085901	0.0218641	-2.957992	-0.632668	0.206002
96	Oman	0	0	0	0		9.376956				
97	Pakistan	1	0	1	0	6.705639	7.385851	0.032618	-2.226336	0.2103905	-0.00742

98	Philippi	1	0	0	1	7.300473	7.566311	0.0351759	-1.868308	0.2756632	0.767238
99	Saudi Ar	0	0	0	0	8.508556	9.277719	0.0462091	-2.647788	-1.407886	0.587411
S-H code	Country	Sample	OECD	Above	Below	ln(GDP60)	ln(GDP85)	n	ln(s)	ln(k/y60)	ln(k/y85)
100	Singapor	1	0	0	1	7.65681	9.191668	0.0301512	-1.182412	0.4391687	1.043625
101	Sri Lank	1	0	1	0	7.374002	7.812783	0.0255817	-2.467823	0.2675692	0.772577
102	Syria	1	0	0	1	7.613325	8.628914	0.0336058	-1.870803	0.3270105	0.466664
103	Taiwan	0	0	0	0	7.393878	8.763271	0.0351314	-1.521497	-0.129887	0.652588
104	Thailand	1	0	0	1	7.107426	8.009363	0.0357858	-1.777629	-0.317957	0.21566
105	United A	0	0	0	0		10.05895				
106	Yemen, N	0	0	0	0		7.628518				
107	Austria	1	1	0	0	8.662678	9.414994	0.0036522	-1.354349	0.5669551	1.057391
108	Belgium	1	1	0	0	8.736811	9.43076	0.0045993	-1.420727	0.7129812	0.939588
109	Cyprus	1	0	1	0	7.821643	8.913416	0.0105084	-1.274203	1.383907	1.012683
110	Denmark	1	1	0	0	8.95377	9.568364	0.0059199	-1.332972	0.7737525	0.999242
111	Finland	1	1	0	0	8.738575	9.498222	0.007799	-1.03911	1.098058	1.182014
112	France	1	1	0	0	8.810907	9.52252	0.0101462	-1.292104	0.6138431	1.000173
113	Germany,	1	1	0	0	8.903135	9.510297	0.005095	-1.252705	0.7996023	1.031066
114	Greece	1	1	0	0	7.789041	8.850087	0.0071363	-1.34737	0.3636037	0.89248
115	Hungary	0	0	0	0		8.685585				
116	Iceland	0	0	0	0	8.700181	9.553504	0.0172199	-1.217266	0.8170886	0.882185
117	Ireland	1	1	0	0	8.273336	9.0524	0.0108567	-1.368908	0.6989613	1.140949
118	Italy	1	1	0	0	8.559103	9.389992	0.0065629	-1.248005	0.8516836	0.968238
119	Luxembou	0	0	0	0	9.090092	9.578381	0.0072853	-1.206026	1.212885	1.002245
120	Malta	0	0	0	0	7.428333	8.705994	0.0092621	-1.438236	0.8636981	0.855757
121	Netherla	1	1	0	0	8.874728	9.45681	0.0143325	-1.378784	0.7547604	0.867471
122	Norway	1	1	0	0	8.770749	9.663261	0.0068962	-1.140757	1.163082	1.131713
123	Poland	0	0	0	0		8.472196				
124	Portugal	1	1	0	0	7.690743	8.662332	0.0061621	-1.441157	0.4756589	0.884202
125	Spain	1	1	0	0	8.194229	9.056956	0.0091739	-1.379395	0.7252532	1.009171
126	Sweden	1	1	0	0	9.051345	9.602315	0.0031392	-1.439371	0.7322655	0.831964
127	Switzerl	1	1	0	0	9.275191	9.699104	0.0086533	-1.25284	0.8049649	1.203034
128	Turkey	1	1	0	0	7.622175	8.232706	0.0290251	-1.559732	0.2331272	0.731985
129	United K	1	1	0	0	8.951699	9.429556	0.0033485	-1.712024	0.3455898	0.656644
130	Yugoslav	0	0	0	0	7.725771	8.67778	0.0122705	-1.203845		
131	Australi	1	1	0	0	9.122602	9.642057	0.0210126	-1.236284	0.9703156	1.003484
132	Fiji	0	0	0	0	7.927324	8.302018	0.0331234	-1.687815	0.777328	0.894295
133	New Zeal	1	1	0	0	9.161885	9.473781	0.0179166	-1.403675	0.5276161	0.886101
134	Papua Ne	1	0	1	0	7.344719	7.622664	0.0247786	-1.84833	0.6153975	1.03761
135	Solomon	0	0	0	0		7.676474				
136	Tonga	0	0	0	0		7.78239				
137	Vanuatu	0	0	0	0		7.763871				
138	Western	0	0	0	0		7.686621				

Note. S-H code:Numerical country code in Summer and Heston data set, ln(GDP60):log of GDP per working-age person in 1960, ln(GDP85):log of GDP per working-age person in 1985, n:Average rate of growth of the working-age population, ln(s): log of average share of real investment,

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TABLE 1
OLS Estimation of The Solow Model
(*Steady State Dynamics*)

Dependent variable: (log difference GDP per working-age person 1960-1985) – 0.5

Sample:	OECD	Above	Below
Observations:	22	32	41
ln(*k/y* 60)	-0.601	-0.451	-0.185
	(0.199)	(0.127)	(0.203)
ln*s*	1.137	0.572	0.662
	(0.285)	(0.099)	(0.230)
ln(*n*+*g*+*d*)	-0.849	-0.564	-0.576
	(0.188)	(0.121)	(0.197)
Raw R^2	0.711	0.647	0.309
Restricted regression:			
ln(*k/y* 60)–ln*s*+ ln(*n*+*g*+*d*)	-0.443	-0.540	-0.088
	(0.078)	(0.075)	(0.148)
Raw R^2	0.608	0.623	0.009
F statistic (test of restriction)	3.36	0.956	8.236
	~ F(2,19)	~F(2,29)	~F(2,38)
Implied *a*	0.31	0.351	0.080

Note: standard errors are in parentheses. *s*, *k/y* 60, and n are the average share of real investment in real GDP for the period 1960-85, the capital-output ratio in 1960, and the average rate of growth of the working-age population for the period of 1960-85, respectively. *g* and *d* are assumed to be 0.02 and 0.05, respectively.

TABLE 2

The Generalized *b* Value in 1985

Sample:	OECD	Above	Below
Observations:	22	32	41
Mean	0.067	0.048	0.063
Standard Deviation	0.006	0.023	0.014
Minimum Value	0.054	0.006	0.028
Maximum Value	0.08	0.083	0.091

Table 3

D. OLS Estimation of The Solow Model

(*Convergence Tests*)

Dependent variable: (log difference GDP per working-age person 1960-1985) – 0.5

Sample:	OECD	Above	Below
Observations:	22	32	41
Constant	2.214	1.260	1.474
	(0.531)	(0.284)	(0.432)
ln(k/y 60)	-0.575	-0.428	-0.257
	(0.207)	(0.128)	(0.199)
ln$s$	1.202	0.551	0.732
	(0.322)	(0.100)	(0.219)
R^2	0.4315	0.515	0.351
Restricted regression:			
Constant	1.423	1.260	0.664
	(0.468)	(0.288)	(0.462)
ln(k/y 60) – ln$s$	-0.6023	-0.536	-0.342
	(0.230)	(0.101)	(0.236)
R^2	0.255	0.484	0.051
F statistic (test of restriction)	5.897	1.838	17.596
	~F(1,19)	~F(1,29)	~F(1,38)
Implied a	0.376	0.349	0.255
Implied b	0.059	0.061	0.107

Note: standard errors are in parentheses. s and k/y 60 are the average share of real investment in real GDP for the period 1960-85 and the capital-output ratio in 1960, respectively.

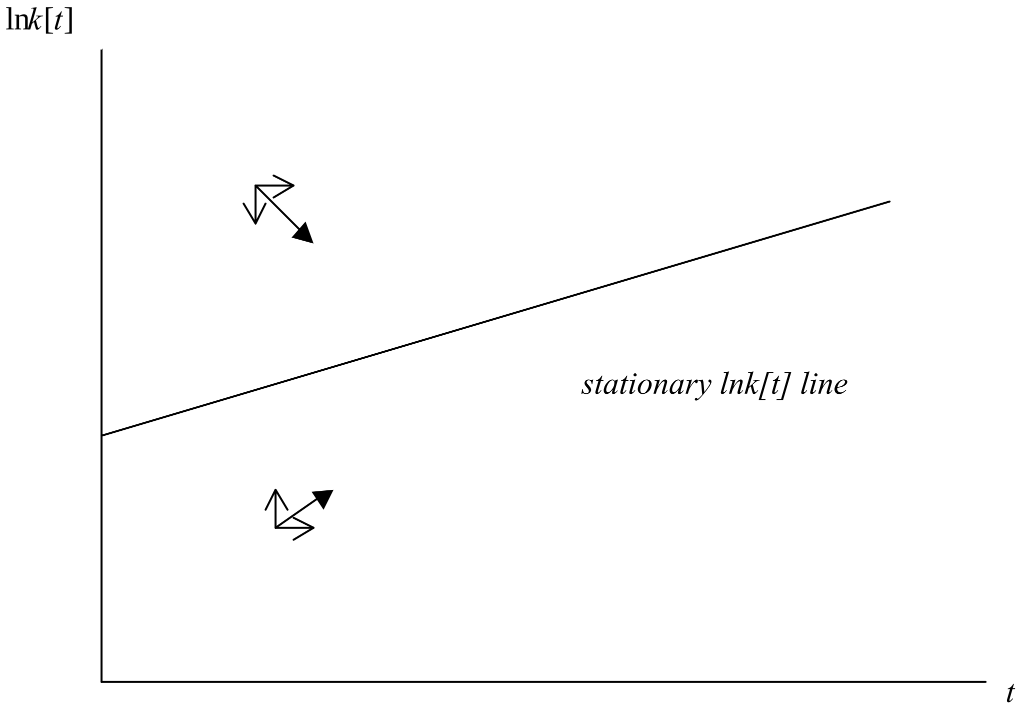

Figure 1 The dynamics of ln$k$[t] over time

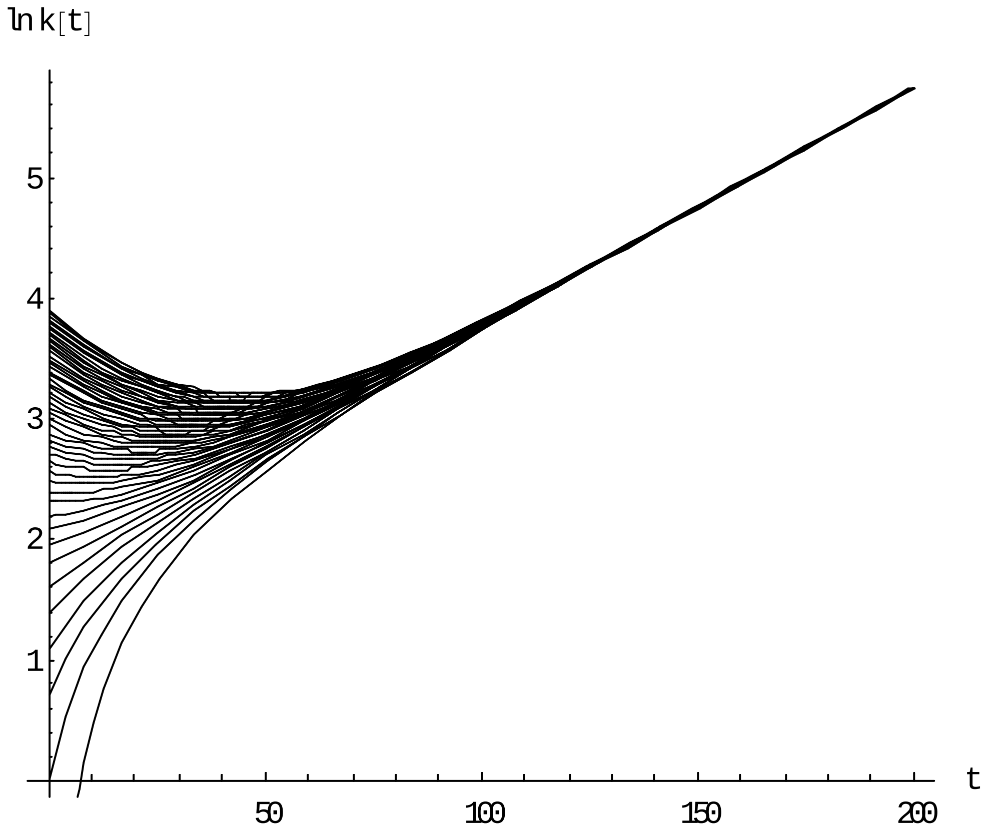

Figure 2 The movement of ln$k$[t] over time

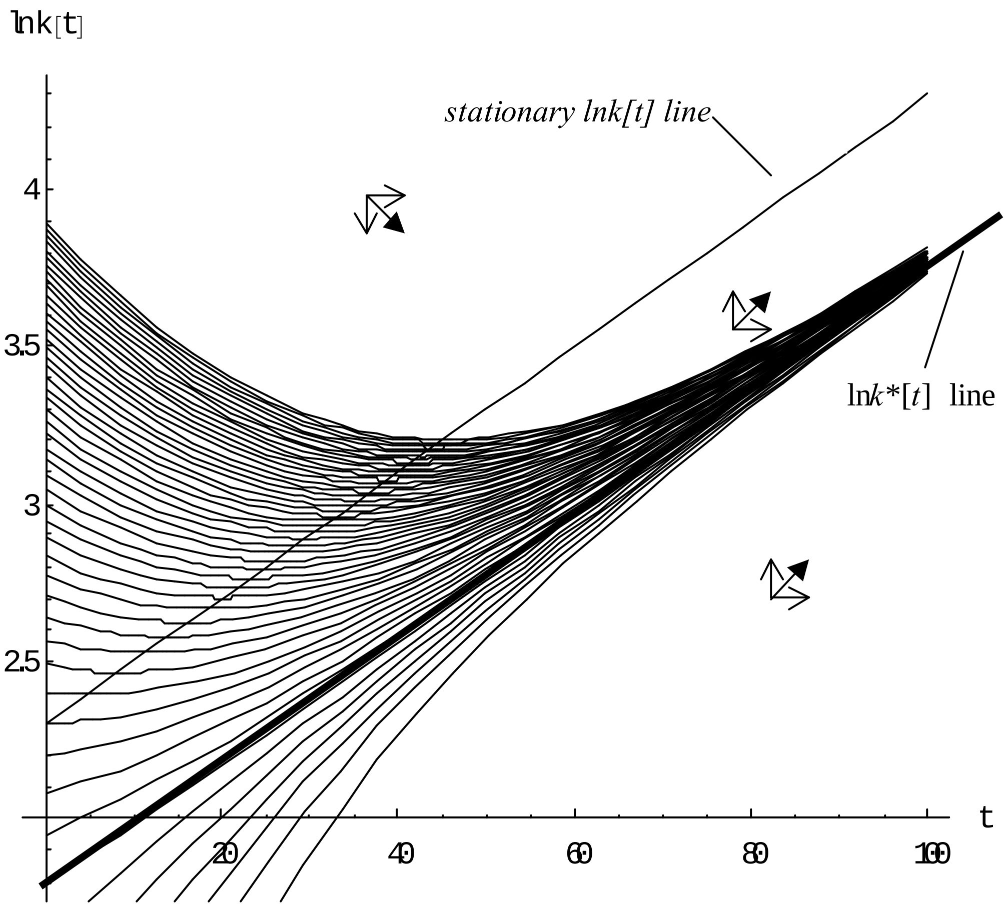

Figure 3 The balanced growth path

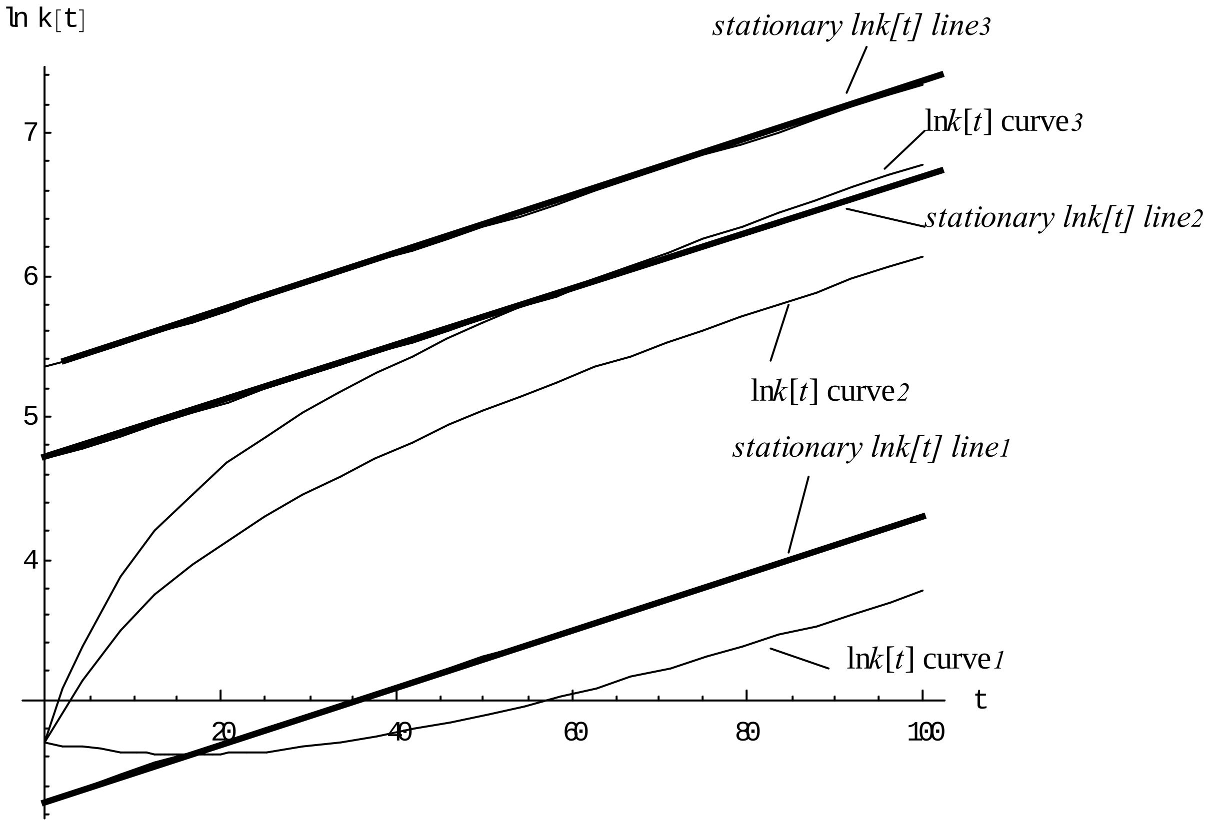

Figure 4 The impact of the difference in $A[0]$

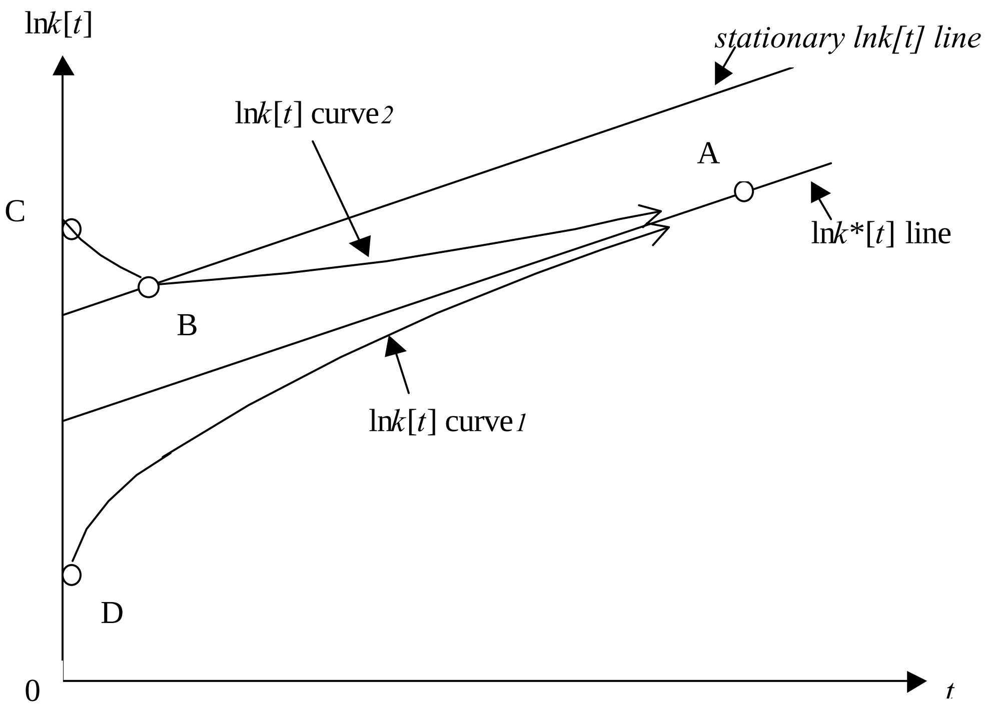

Figure 5 Three kinds of paths of ln*k*[*t*]

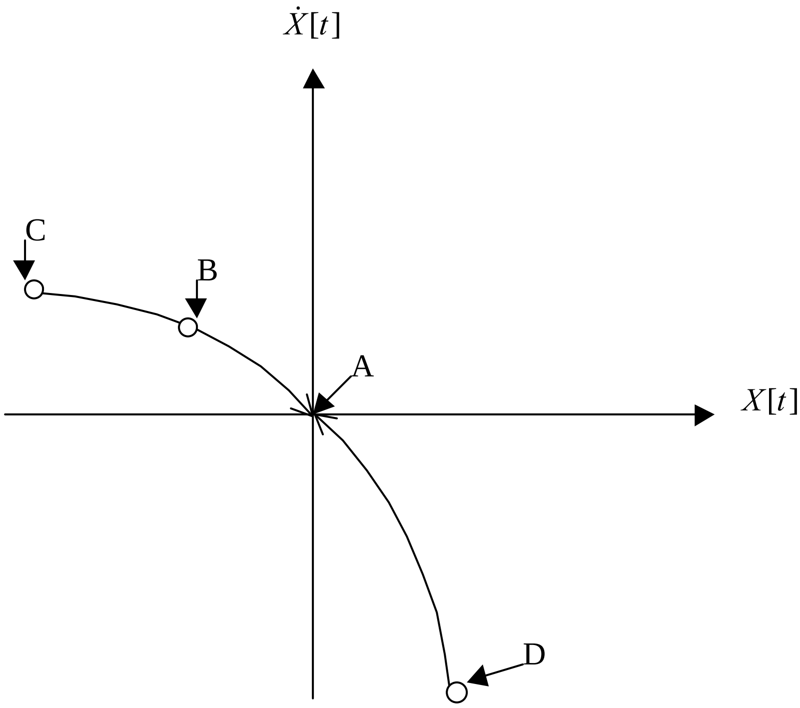

Figure 6 *b* convergence

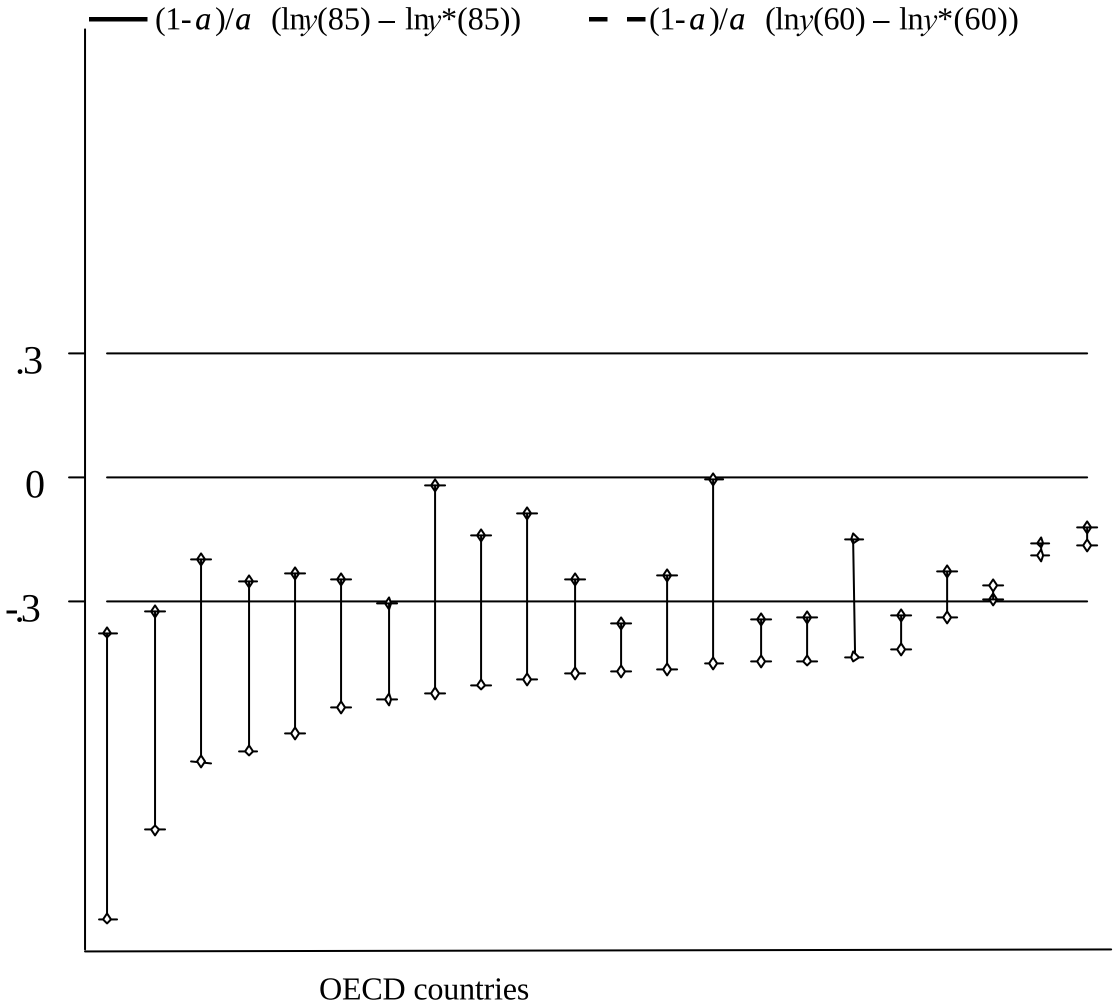

Figure 7 Overshooting of the steady state (OECD)

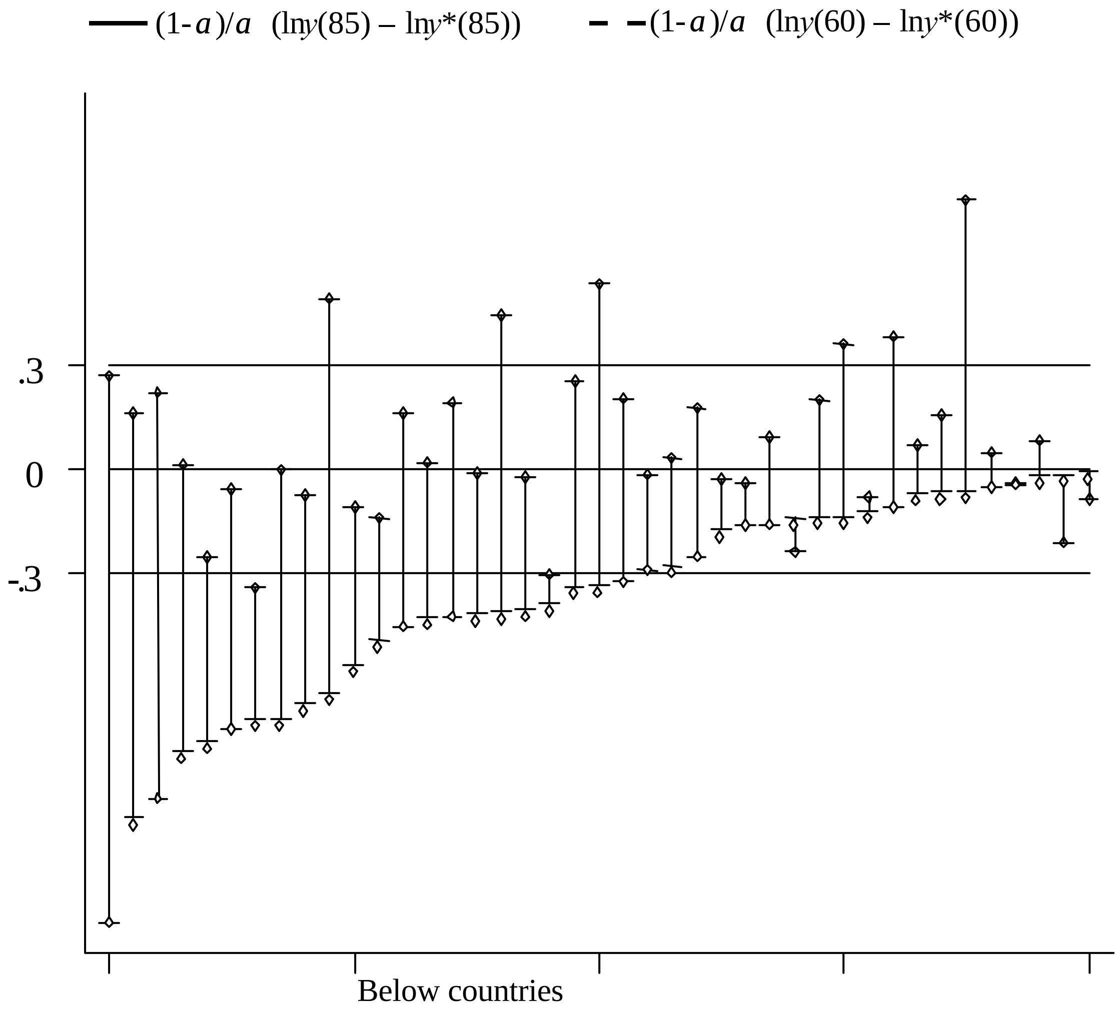

Figure 8 Overshooting of the steady state (Below countries)

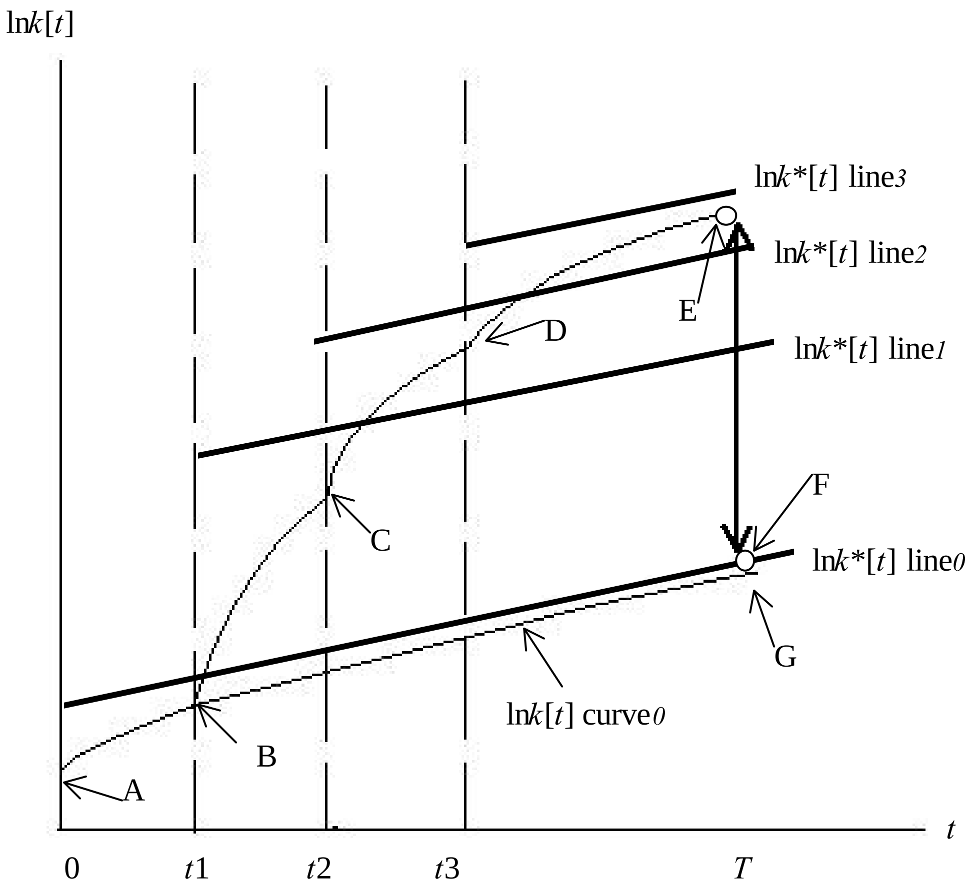

Figure 9 Overshooting

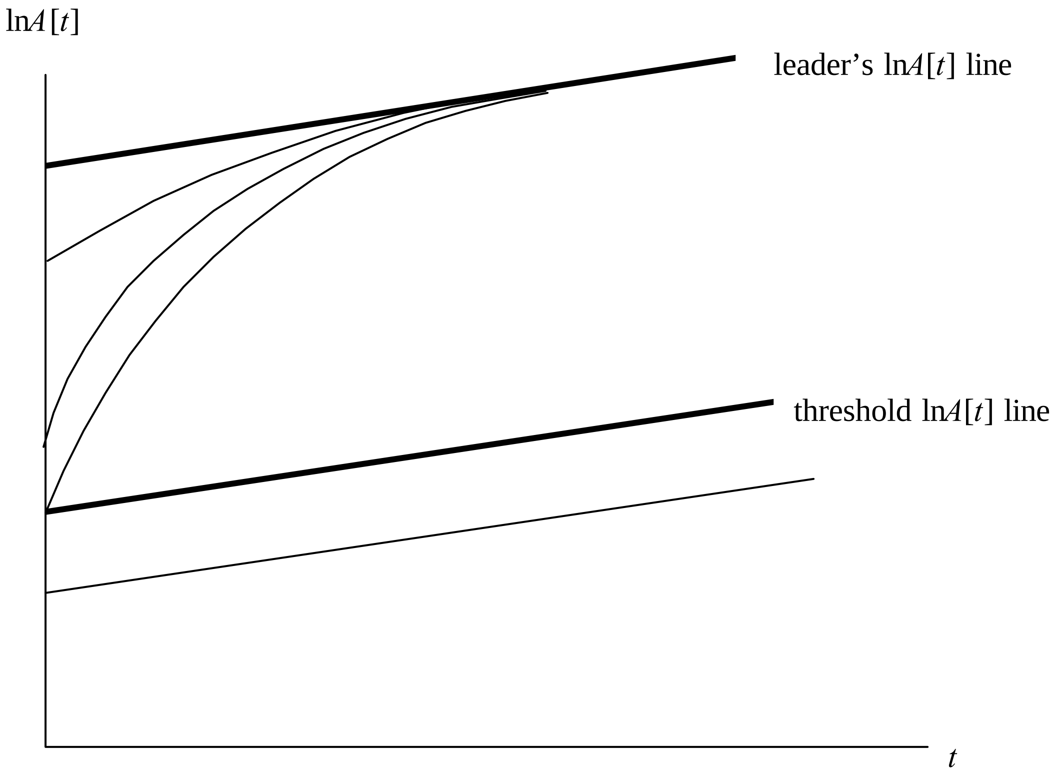

Figure 10 Diffusion of technology