REFORM, LOBBIES AND WELFARE: A COMMON AGENCY APPROACH

Cecilia TESTA
London School of Economics - STICERD
Houghton street WC2A 2AE London
e-mail: C.Testa@lse.ac.uk

Abstract

Policy makers make policy decisions, which affect the utility of private citizens. The traditional explanation for government intervention in the economy is the existence of market failure. Nevertheless, despite public policy interventions, the economy may fail to reach the efficient frontier. Hence, unless we suppose that efficient policies are not feasible (second and third best arguments), we need a theory of inefficient decisions by policy makers. Therefore the following questions arises: How do policy makers take policy decisions? What is their objective function? The objective of this paper is to verify if policy-makers' preferences for monetary transfers can generate inefficiency. We analyse a policy making process where the policy decisions are a *reform* and a *compensating taxation*. The main feature of the reform is that it creates gainers and losers. Redistributive taxation can be used to compensate losers. Formally, the relationship between the citizens and the policy maker is modeled as a *common agency game*, where the citizens, organized in lobby groups, are the principals and the policy maker is the agent. With the possibility for at most two lobbies, we find that one-lobby equilibrium is inefficient and the two-lobbies equilibrium is efficient. When we check for the Nash equilibrium we find that, non-lobby and one-lobby are not Nash equilibria and two-lobbies is a Nash equilibrium. Finally, the unique Nash equilibrium is robust to the introduction of a political constraint on tax parameters.

keywords: reform, lobbying, redistribution, economic welfare.

Introduction

Policy makers make decisions which affect the utility of private citizens. The traditional explanation for government intervention in the economy is that the market equilibrium may be inefficient and therefore government intervention can correct market failure. Nevertheless, it is not obvious that the policy maker's intervention restores efficiency. In fact, we can find examples of inefficiency despite government intervention. In this sense, a particular interesting case is the transition of Eastern European countries from central planning to market regimes. The transition process involves many structural reforms such as price liberalization, restructuring and privatization. Even though it is generally agreed that such reform should be implemented, it is often the case that reforms are partially implemented or not undertaken at all.

Therefore we can ask why efficient reforms are delayed or not undertaken. An explanation for this type of inefficiency is that efficient policies may not be feasible (second best and third best arguments). However, although the problem of the effective existence of policy instruments is important, other factors could be responsible for this inefficiency. For example, the policy making process might be structured in a way that generates inefficient policy decisions. This problem is not taken into account by traditional welfare economics. In this sense, the political economy approach, focusing on the process of policy decision making rather than assuming a benevolent dictator, could provide an alternative explanation for inefficient policies.

If we exclude that the lack of instruments is the cause of the inefficiency, we need a theory of public decision making where the inefficiency could be related to the decision process. Therefore the following questions arise: *How do policy makers take policy decisions? What is their objective function?* The answer to these questions can help us build a theory of *government failure* and then find other responses to the efficiency question. On a more practical level, these kinds of questions are particularly important to understand the experience of the economies in transition. In fact, the role of government is crucial in the implementation of the reforms that drive the economy from the central planning to the market regime. Therefore, investigating how policy makers take policy decisions is important to understand the reform pattern followed by the transitional countries.

In this paper we analyze public decision-making, focusing on the relationship between the policy maker and the citizens from a theoretical perspective. In democratic systems citizens essentially influence government through voting. However, private transfers to policy makers are another mechanism to orientate policy decisions. Our analysis will focus on this second channel of influence. Our *assumption* is that policy makers are individuals that have preferences like all the other individuals in the society and therefore they do not dislike monetary transfers. The government's preference for transfers implies that citizens can use transfers to obtain their most preferred policy, where the optimal policy for a single citizen needs not to be efficient for the economy. Hence, we can conjecture the following two *statements*. First, the policy-makers' preferences imply lobbying action. Second, given the citizens preferences, lobbying can produce inefficiency. Therefore, the objective of this paper is to verify if these two statements are correct.

In formal terms, we model the relationship between the citizens and a single policy maker as a *common agency game*, where the citizens, organized in lobby groups, are the principals and the policy maker is the agent. We employ a *theoretical framework* similar to that of Dixit-Grossman-Helpman (1996). We consider, as a particular example of policy making, a reform process where the relevant policy decisions are a *reform* and a *compensating taxation*. The reform we analyze is *price liberalization*. Before the reform takes place, prices are below the market equilibrium level. Therefore the *status quo* regime is characterized by rationing. The individuals in the economy are heterogeneous in their income. Hence, depending on their nominal income, the consumers in this economy can be divided in two groups: rationed and non-rationed.

The main feature of the reform is that it creates gainers and losers because the rationed consumers gain from the reform and the non-rationed loose. Non distortive redistributive taxation can be used to compensate losers. Hence, there are two policy variables, i.e. reform and taxation, and two groups that have opposite interests. The two groups can organize themselves into two lobbies and they can offer payments to the policy-maker in order to obtain their most preferred policy. Their payment function is *truthful* in the sense that it truthfully represents the individuals' preferences for the policy. We look for an equilibrium in truthful strategies of the common agency game and, in particular, we are concerned with the efficiency properties of that equilibrium. We find that the one-lobby equilibrium is inefficient and the two-lobbies equilibrium is efficient. When we check for the Nash equilibrium we find that the non-lobby and the one-lobby are not Nash equilibria and two-lobbies is a Nash equilibrium. Finally, the unique Nash equilibrium is robust to the introduction of a political constraint on tax parameters.

Therefore, we conclude that if all citizens can use monetary transfers to influence the government, then public decision making does not generate inefficiency because the truthful revelation of individual preferences implies that the social surplus maximizing policy is chosen. On the other hand, if some groups are excluded from lobbying, then inefficient policies may be chosen.

The plan of the paper is as follows. In *section 1* we describe the general theoretical framework. In *section 2* we characterize the equilibrium of the lobby game. In *section 3* we analyze the efficiency properties of the lobby and non-lobby equilibria. In *section 4* we analyze the lobbying decision of the special interest groups in the lobby game and we discuss the main results. In *section 5* we summarize and conclude.

1 General theoretical framework

The economy consists of n individuals, $(n-1)$ citizens indexed i, and a policy maker indexed j. The pre-reform regime is characterized by fixed prices and rationing, i.e. the level of prices is too low, hence the demand side is rationed. Rationing is uniform, which means that each individual can only access the same fixed amount of the rationed good. We assume that individuals are heterogeneous with respect to their income, therefore the actual rationing they perceive is differentiated since rich individuals are more rationed than poor individuals. Given their income level, we divide the individuals into two types, rationed and non-rationed , indexed respectively R and NR. The number of rationed is n_R and the number of non-rationed is n_{NR}. The policy maker is either rationed or non-rationed.

Price liberalization is necessary to eliminate rationing. When the reform is implemented, given the excess of demand of the pre-reform regime, prices increase to clear the market. In the walrasian equilibrium we will observe higher prices and higher output. If we represent the economy in terms of utility possibility set, price reforms shifts outwards the pareto frontier, therefore the reform is efficient. On the other hand, the reform is not pareto-improving. Since not all the individuals are actually rationed, there is a group of gainers from liberalization (rationed) and a group of losers (non-rationed). We assume that lump sum transfers can be used to compensate losers, therefore compensating redistribution is part of the reform package. Then it is clear that there are *opposite interests* on the reform package. The rationed citizens gain from reform but lose from redistribution, for the non-rationed the opposite is true. The two types of citizens organize in two lobby groups and try to obtain their preferred reform package offering the policy maker a payment that is a function of the policy. The policy consists of a price liberalization parameter, δ, and a lumps sum transfer, T. Each citizen i offer the policy maker j a payment $C_i(\delta, T)$. The policy maker then chose the optimal policy, given the payments of all the citizens. To summarize, the citizens and the policy maker have their own objective functions depending on the policy vector, (δ, T) and on the payment function, $C_i(\delta, T)$. The reform package chosen at the equilibrium is the outcome of the lobby game where the citizens and the policy maker are maximizing their respective objective functions. In what follows we define the objective functions, we characterize formally the reform process and we describe in more details the lobby game.

1.1 Objective Functions

Let's denote i the citizens and j the policy maker. The citizens are rationed and non-rationed, the policy maker is either rationed or non-rationed. We use the subscript R for the rationed and NR for the non-rationed. The number of rationed is n_R and the number of non-rationed is n_{NR} . The reform parameter is denoted δ, the lump sum tax is T and the lobby payment from citizen i to the policy maker j is $C_i(\delta, T)$.

The reform parameter assumes values between zero and one, the transfer parameter takes values between a positive minimum value denoted $T_{\min}$ and a strictly positive maximum value denoted $T_{\max}$, the lobby payment belongs to the set of the *feasible payments* that we denote $\mathcal{C}_i$. We also assume that the public budget is balanced.

The utility of both citizens and policy maker depends on the reform parameter, the level of transfers and the lobby payment they make or receive (depending on the fact that they are citizens or policy maker). The utility function is quasi-linear[1]: concave in the reform parameter and linear in money. Depending on their identity, the individuals can gain or lose from reform: rationed agents gain from reform (utility increasing in δ) while non-rationed lose from it (utility decreasing in δ). The opposite is true for transfers: the non-rationed's utility is increasing in transfers while the rationed's utility is decreasing. Finally the lobby payment decreases the utility of the citizens and increases the utility of the policy maker. In formal terms, the objective functions of the citizens and the policy maker are expressed by the following indirect utility functions:

$$U_{i=R}\left[\delta, T, C_R\left[\delta, T\right]\right] = V_{i=R}\left(\delta\right) - \alpha T - C_{i=R}\left[\delta, T\right] \tag{1}$$

$$U_{i=NR}\left[\delta, T, C_{NR}\left(\delta, T\right)\right] = V_{i=NR}\left(\delta\right) + T - C_{i=NR}\left[\delta, T\right] \tag{2}$$

$$G_{j_{|j=\{R,NR\}}}\left[\delta, T, C_{NR}\left[\delta, T\right], C_R\left[\delta, T\right]\right] = V_j\left(\delta\right) + \alpha_j T + (n_{NR} - l_j) C_{NR}\left[\delta, T\right] + (n_R - 1 + l_j) C_R\left[\delta, T\right] \tag{3}$$

where
$0 \leq \delta \leq 1$
$0 \leq T_{\min} \leq T \leq T_{\max}$
$C_i\left(\delta, T\right) \in \mathcal{C}_i$
$n_{NR} + n_R = n$
$\alpha = \frac{n_{NR}}{n_R}, \alpha_j = 1$ if $j = NR, \alpha_j = -\alpha$ if $j = R$
$l_j = 1$ if $j = NR,\ l_j = 0$ if $j = R$
$V_i'\left(\delta\right) > 0$ iff $i = R, V_i''\left(\delta\right) < 0$
$V_j'\left(\delta\right) < 0$ iff $j = NR, V_j''\left(\delta\right) < 0$

[1] caveat on quasi-linearity: this assumption simplifies our analysis so that we can we easily derive the equilibrium policy of the lobby game. On the other hand, we do not derive interesting results on transfers because, as we will see, quasi-linearity implies that the social surplus does not depend on transfers. Nevertheless, since our main objective is to discover if lobbies distort the reform parameter, we think that assuming quasi-linearity doensn't imply a loss of generality.

1.2 Characterization of the reform

Price liberalization is summarized by the parameter δ, where $\delta = 1$ stays for full reform, $\delta = 0$ no reform and $0 < \delta < 1$ partial reform. Concerning transfers, the minimum and the maximum levels, $T_{\min}$ and $T_{\max}$, depend respectively on the reservation utility of the non-rationed and rationed individuals: the minimum level of transfers the non-rationed are disposed to accept has to guarantee their reservation utility, $\bar{u}_{NR}$; the maximum level the rationed are disposed to pay has to guarantee their reservation utility, $\bar{u}_R$. Furthermore, we assume that the reservation utility of the non-rationed individuals, $\bar{u}_{NR}$, is less or equal to their *status quo* utility, u_{NR}^{SQ}, and the reservation utility of the rationed, $\bar{u}_R$, is greater or equal to their *status quo* utility, u_R^{SQ}. Finally, we assume that, when full reform is implemented, the maximum level of transfers is sufficient to guarantee the non-rationed the *status quo* utility while, in the same case, the minimum level of transfers is not sufficient. We also assume that the rationed individuals prefer full reform with the maximum level of transfers to partial reform with a lower level of compensation[2]. Formally our assumptions on transfers are the following:

- **Assumption 1:**

$$T_{\min} : U_{NR}\left[\delta, T_{\min}, C_{NR}(\delta, T)\right] \geq \bar{u}_{NR}, \quad \bar{u}_{NR} \leq u_{NR}^{SQ} = V_{NR}(0) \tag{4}$$

$$T_{\max} : U_R\left[\delta, T_{\max}, C_R(\delta, T)\right] \geq \bar{u}_R, \quad \bar{u}_R \geq u_R^{SQ} = V_R(0) \tag{5}$$

$$U_{NR}\left[1, T_{\max}, C_{NR}(\delta, T)\right] \geq u_{NR}^{SQ} > U_{NR}\left[1, T_{\min}, C_{NR}(\delta, T)\right] \tag{6}$$

$$U_R\left[1, T_{\max}, C_R(\delta, T)\right] > U_R\left[\delta, T, C_R(\delta, T)\right] \tag{7}$$

The equation 4 says that the non-rationed agents can lose from reform ($\bar{u}_{NR} \leq u_{NR}^{SQ}$) but not indefinitely ($U_{NR}\left[\delta, T, C_{NR}(\delta, T)\right] \geq \bar{u}_{NR}$). The equation 5 says that the rationed individuals are disposed to pay compensation for reform to the non-rationed up to the point they get the *status quo* utility (pareto improving redistribution). The inequality 6 says that full reform with the maximum level of transfers is pareto-improving, while full reform with the minimum level of transfers is not. Inequality 7 says that the gain from full reform is sufficient to give the rationed individuals higher utility than partial reform, whatever the level of compensation they have to pay. Note that full reform with minimum transfers is the most preferred policy of the rationed individuals (that gain from reform but lose from redistribution) and is

[2] This means that gains from full reform are high enough to make the rationed individuals better off even if they have to pay the maximum level of compensation. This assumptions seems appropriate when the rationing is particulary severe so that the loss from partial reform is important.

the worst outcome for the non-rationed ones (that lose from reform but gain from redistribution). Therefore, the two groups of individuals have opposite interest on the reform package. The rationed agents prefer full reform, $\delta = 1$, to partial reform, $\delta < 1$, for every level of compensation they have to pay to the non-rationed. Obviously they prefer full reform with the lowest compensation to full reform with the highest compensation. Concerning the non-rationed agents, we have that, since they lose from reform, they prefer partial reform to full reform when the minimum level of compensation is paid. But they are better off with full reform if the maximum level of compensation is paid[3].

1.3 Political Power

Citizens have political power if, independently of lobby contributions, they can enforce positive compensation when they suffer a loss from reform. In this model the citizens that lose from reform are the non-rationed individuals. The utility they get from reform is $V_{NR}(\delta)$. Enforcing compensation means that, independently of lobby payment, the minimum level of transfers must be strictly positive, $T_{\min} > 0$. An example of how losers can "enforce" positive compensation could be the threat of social unrest if redistribution doesn't occur.

Formally, the assumptions on the political power are the following:

• **Assumption 2A**: *no political power for NR*:

$V_{NR}(\delta) > \bar{u}_{NR} \; \forall \delta > 0$

• **Assumption 2B**: *political power for NR*:

$V_{NR}(\delta) \leq \bar{u}_{NR} \leq V_{NR}(0)$

assumption 2A says that non-rationed agent cannot enforce compensation, assumption 2B says that they can enforce compensation[4].

1.4 Lobby game

The rationed and the non-rationed citizens organize in lobbies, therefore we have two lobby groups. The policy maker can be either a rationed or a non-rationed and the

[3] The utility ranking on reform packages is the following:
$V_{NR}(1) + T_{\max} > V_{NR}(\delta) + T_{\min} > V_{NR}(1) + T_{\min}$
$V_R(1) - \alpha T_{\min} > V_R(1) - \alpha T_{\max} > V_R(\delta) - \alpha T$

[4] Note that from assumption 1, equation 4, and assumptions 2A and 2B we have that:

• if $V_{NR}(\delta) > \bar{u}_{NR} \; \forall \delta > 0 \implies T\min = 0$ satisfies equation 4, therefore $T\min > 0$ is not necessary.

• if $V_{NR}(\delta) \mid_{\delta \neq 0} \leq \bar{u}_{NR} \leq V_{NR}(0) \Rightarrow T\min = 0$ violates equation 4, therefore $T\min > 0$ is necessary.

identity of the policy maker is exogenously given[5].

The policy maker chose the policy (reform and compensatory transfers) that affects the utility of all the individuals in the society and those individuals try to influence the policy maker's decision trough lobbying. In particular, the relationship between the lobbies and the policy maker is modeled as a *common agency game* where the two lobbies are the two principals and the policy maker is the agent. In the first stage, the principals offer the agent a policy and a payment, simultaneously and non-cooperatively. In the second stage, the policy maker chose the optimal policy, given the payment functions of all the principals. We assume that the payment functions are non-negative, $C_i(\delta, T) \geq 0$ and globally truthful, which means that the citizens are available to pay their true evaluation of the policy[6].

2 Lobby game: characterization of the equilibrium

The objective of this section is to analyze how lobbying affects policy. For this, we have to derive the lobby equilibrium and then compare it with the non-lobby equilibrium to see if lobbies have an effect on policy decisions.

Therefore, in what follows, we first define the lobby equilibrium as the equilibrium of the common agency game, where the lobbies are the principals and the policy-maker is the agent; then we use two fundamentals results of Dixit-Grossman-Helpman (1996)(hereafter DGH) to characterize the equilibrium. Using the DGH results we can easily solve the lobby game to get the equilibrium policy parameters and we can compare these parameters with the equilibrium parameters of the non-lobby game to discover if lobbying distorts policy. We conduct the analysis under the alternative

[5] Note that, since the identity of the policy maker is exogeneously given, there is not voting and electoral competition in this model. Thus the focus of the analysis is exclusively on the relationship between the policy maker and the citizens and its the objective is to understand how citizens can distort policies trough lobby action, once a goverment is in place. An alternative and more satisfying way to model the identity of the policy maker is the citizen-candidate approach of Besley-Coate (1996), where the policy maker identity is endogeneised.

[6] A payment function $C_i[\delta, T, u_i^*]$ for principal i is globally truthful relative to constant u_i^* if:

$C_R[\delta, T, u_R^*] = \min[\varphi(\delta, T, u_R^*), c]$

$C_R[\delta, T, u_{NR}^*] \equiv \varphi(\delta, T, u_R^*) = V_R(\delta) - \alpha T - u_R^*$

$\frac{\partial C_R}{\partial \delta} = V_R'(\delta)$

$\frac{\partial C_R}{\partial T} = -\alpha$

$C_{NR}[\delta, T] = \min[\varphi(\delta, T, u_{NR}^*), c]$

$C_{NR}[\delta, T] \equiv \varphi(\delta, T, u_{NR}^*) = V_{NR}(\delta) + T - u_{NR}^*$

$\frac{\partial C_{NR}}{\partial \delta} = V_{NR}'(\delta)$

$\frac{\partial C_{NR}}{\partial T} = 1$

Competition in truthful strategies boils down to non-cooperative choices of the constants u_i^* which determine the equilibrium payoffs of the principals.

assumptions on the political power of the lobby groups.

2.1 Lobby Equilibrium

Consider the set of the feasible payments, $\mathcal{C}_i$. The equilibrium of the common agency game from definition of sub-game perfect Nash equilibrium for a two-stage game consists of a vector of payment functions $C^\circ = (C_i^\circ)_{i=(R\cup NR)}$ and a policy vector (δ°, T°) such that:

(a) $C_i^\circ \in \mathcal{C}_i$ $\forall i \in (R \cup NR)$

(b) $(\delta^\circ, T^\circ) = \arg\max G_j \left[\delta, T, C_{NR}^\circ, C_R^\circ\right]$

(c) $\forall i = R, NR$ $\;C_i^\circ$ is a best response of principal i to the payment function of the other principal.

From Dixit-Helpman-Grossman (1996) we know that a vector of payment functions $C^\circ = (C_i^\circ)_{i=(R\cup NR)}$ and a policy vector (δ°, T°) constitute an equilibrium of the common agency game if and only if the following is true:

RESULT 1- DGH

(a) $C_i^\circ \in \mathcal{C}_i$ $\forall i \in (R \cup NR)$

(b) $(\delta^\circ, T^\circ) = \arg\max G_j \left[\delta, T, C_{NR}^\circ, C_R^\circ\right]$

(c) $[\delta^\circ, T^\circ, c^\circ] = \arg\max_{\delta,T,c} U_i^L [\delta, T, c]$ $\forall i = R, NR$ *and*

$G_R\left(\delta^\circ, T^\circ, C_{NR}^\circ, C_R^\circ\right) = \max_{\delta',T'} G_R\left(\delta', T', C_{NR}\left(\delta', T'\right), 0\right)$

When we restrict our analysis to the set of global truthful payment functions, we can use another result of Dixit-Helpman-Grossman (1996) to characterize the equilibrium of the lobby-game. Let $(C_R^\circ, C_{NR}^\circ, \delta^\circ, T^\circ)$ be a truthful equilibrium where $(u_R^\circ, u_{NR}^\circ)$ are the equilibrium payoffs of the principals, then this equilibrium is characterized by 2 conditions:

RESULT 2-DGH

(a) *the action,*(δ°, T°) *, undertaken by the policy-maker (agent) must maximize his objective function. Formally:*

$$(\delta^\circ, T^\circ) = \arg\max G_j \left[\delta, T, C_{NR}^\circ, C_R^\circ\right]$$

(b) *the payment,* C_i°*, made by the 2 lobby groups (principals) must satisfy a participation constraint. This means that each lobby must provide the policy maker at least the same level of utility of his outside option (the level of utility that the policy maker obtains when he plays his best response given that only the other lobby group is offering a positive payment). Formally:*

$$G_R\left(\delta^\circ, T^\circ, C_{NR}^\circ, C_R^\circ\right) = \max_{\delta',T'} G_R\left(\delta', T', C_{NR}\left(\delta', T'\right), 0\right)$$

We can have 2 types of policy-maker (rationed and non-rationed), but for the moment we derive the truthful equilibrium considering only a rationed policy-maker[7]. Since we are assuming global truthfulness, we can use the second result of Dixit-Grossman-Helpman to solve the lobby game and derive the truthful equilibrium.

The following proposition gives a characterization the lobby equilibrium under the alternative assumptions of political power of the non-rationed individuals.

Let $(\delta^{\circ}, T^{\circ}, C_{NR}(\delta^{\circ}, T^{\circ}, u^{\circ}_{NR}), C_R(\delta^{\circ}, T^{\circ}, u^{\circ}_R))$ be a truthful equilibrium where $(u^{\circ}_R, u^{\circ}_{NR})$ are the payoffs of the principals, then:

Proposition 1 *The equilibrium liberalization parameter,*δ°, *is the social surplus maximizing parameter and the equilibrium transfer,* T°, *is* $T^{\circ} = 0$ *under the assumption of no political power and* $T^{\circ} = T_{\min} > 0$ *under the assumption of political power.*

Proof.

To prove proposition 1 we use the result 1 from DGH. The first condition characterizing the lobby equilibrium says that the equilibrium policy chosen by the policy maker must maximize his objective function. Therefore, $(\delta^{\circ}, T^{\circ})$ must satisfy the following condition:

$$(\delta^{\circ}, T^{\circ}) = \arg\max_{\delta, T} V_R(\delta) - \alpha T + n_{NR} C_{NR}[\delta, T, u^{\circ}_{NR}] + (n_R - 1) C_R[\delta, T, u^{\circ}_R]$$

$$\text{st. (c)} \qquad T - C_{NR}(\delta, T, u_{NR}) \geq \bar{u}_{NR} - V_{NR}(\delta)$$

where (c) is the political constraint

Note that if we use the definition of the payment functions, $C_i[\delta, T, u^{\circ}_i]$, and we replace it in the objective function of the policy maker, then this objective function turns out to be just the social surplus:

$$G_R[\delta, T] = n_R V_R(\delta) + n_{NR} V_{NR}(\delta) - K$$

where, $K = n_{NR} u^{\circ}_{NR} + (n_R - 1) u^{\circ}_R$.

Solving this maximization problem with and without political constraint we obtain the social surplus maximizing value for the reform parameter, while for the tax parameter, under the assumption of no political power it is not an argument of the objective function of the policy maker, hence transfers are not paid. Under

[7] We will see that the identity of the policy maker doesn't have any effect on the truthful equilibrium of the 2-lobbies game. The assumption on the identity becomes important in the second part of our analysis, that is when we analyse the lobbying decision of the lobby group in the lobby game. In this decision process the identity of the policy maker is important because it determines the payoff of the non-lobby equilibrium which affects on the lobbying decision.

the assumption of political power, the tax parameter is determined by the political constraint that we prove to be binding[8] (see appendix).

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To summarize, the equilibrium liberalization parameter of the lobby game is just the social *surplus maximizing value* and this value also satisfies the participation constraint of the common agency game[9]. Concerning the tax parameter, it depends on the assumption on the political power.

2.2 Non-lobby equilibrium

We characterize in this section the non-lobby equilibrium and then we will compare it with the lobby equilibrium to see how lobby affect policy.

Let $j \in \{R, NR\}$ be the policy maker identity and $\left(\delta_j^{NL}, T_j^{NL}\right)$ be the equilibrium policy vector of the non-lobby game chosen by the policy maker j :

Proposition 2 *When the policy maker is a non-rationed individual the equilibrium policy vector is* $\left(\delta_{NR}^{NL} = 1, T_{NR}^{NL} = T_{\max}\right)$. *When the policy maker is a rationed individual the equilibrium policy vector is* $\left(\delta_R^{NL} = 1, T_R^{NL} = 0\right)$ *under the assumption of no political power and* $\left(\delta_R^{NL} = 1, T_R^{NL} = T\min > 0\right)$ *under the assumption of political power.*

The proof of this result is straightforward. We know that the preferred outcome of a rationed individual is full reform with the minimum level of compensation. The non-rationed prefers full reform with the maximum level of compensation. Therefore, depending on the identity of the policy maker, we have 2 equilibria and both are *efficient* (full reform), the difference between the two equilibria is only in the transfer parameter, T: a non-rationed policy maker sets the transfer to the maximum level, while a rationed policy maker pays the minimum level of transfers, where this minimum level is zero when the non-rationed cannot enforce positive compensation (they do not have "political power") and is strictly positive when the rationed have political power.

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[8] replacing the binding the constraint in the objective function and solving the maximization, by the first order condition we obtain:

FOC: $n_R V_R'(\delta) + n_{NR} V'(\delta) = 0 \Rightarrow 0 \leq \overset{\sim NL}{\delta}_R \leq 1$

[9] The participation constraint (condition (b) of the RESULT 2-DHG) is expressed by the following equations (see appendix for formal proof):

$(\text{b})_{\text{R}}$ $(n_R - 1) V_R(\delta^\circ) + [V_R(\delta^\circ) - V_R(\delta_{NR}^\circ)] + n_{NR}[V_{NR}(\delta^\circ) - V_{NR}(\delta_{NR}^\circ)] - (n_R - 1)\alpha T\max = (n_R - 1) u_R^\circ$

$(\text{b})_{\text{NR}}$ $n_R [V_R(\delta^\circ) - V_R(\delta_R^\circ)] + n_{NR} V_{NR}(\delta^\circ) + n_{NR} T\min = n_{NR} u_{NR}^\circ$

2.3 Comparing lobby and non-lobby equilibria

In this section we compare lobby and non-lobby equilibria to verify if lobbies distort policies. Since we characterized the equilibria under alternative assumption on the political power of the rationed individuals, before comparing the lobby and non-lobby equilibria, let's define the following notation for the equilibria with an without political constraint:

$\left(\delta_j^{NL}, T_j^{NL}\right)$ = equilibrium of the *unconstrained non-lobby game* (NL) when the policy maker is j.

$(\delta^{\circ}, T^{\circ}, C_i^{\circ}, u_i^{\circ})_{i=NR,R}$ = equilibrium of the *unconstrained 2-lobby game.*

$(\delta_i^{\circ}, T_i^{\circ})$ = parameters chosen in the *unconstrained lobby game* when only group i is lobbying .

$\left(\tilde{\delta}_j^{NL}, \tilde{T}_j^{NL}\right)$ = equilibrium of the *constrained non-lobby game* (NL) when the policy maker is j.

$\left(\tilde{\delta}^{\circ}, \tilde{T}^{\circ}, \tilde{C}_i^{\circ}, \tilde{u}_i^{\circ}\right)_{i=NR,R}$ = equilibrium of the *constrained 2-lobby game*

$\left(\tilde{\delta}_i^{\circ}, \tilde{T}_i^{\circ}\right)$ = parameters chosen in the *constrained lobby game* when only group i is lobbying .

Comparing the non-lobby and the lobby equilibria in the case of *no political constraint*, we can see that in the non-lobby equilibrium we obtain the individual maximizing values of reform and transfers, therefore the identity of the policy maker is relevant: a rationed and a non-rationed policy-maker both implement full reform, but the rationed chose zero transfers while the non-rationed chose the maximum level of transfers. In the lobby equilibrium we obtain the social surplus maximizing value of the reform parameter and this does not depend on the policy maker identity but on the number of rationed versus non-rationed people. Concerning transfers, they just depend on who has the political power. When the non-rationed agents do not have political power (assumption 2A), transfers are not paid in the lobby equilibrium, and this is true even if the policy maker is non-rationed (this is just a consequence of quasi-linear utility function). Note that, since in the lobby equilibrium we do not have transfers, the best the non-rationed individuals can do is just to lower the reform parameter. Indeed if the ratio of rationed versus non-rationed is the following:

$$\frac{n_R}{n_{NR}} < \left(\frac{V_{NR}(\delta) - V_{NR}(1)}{V_R(1) - V_R(\delta)}\right)$$

then the non-rationed can obtain partial reform $(\delta^{\circ} < 1)$[10].

[10]for $\delta^{\circ} = 1$ to be an equilibrium, the following must be true:

$$n_R V_R(1) + n_{NR} V_{NR}(1) > n_R V_R(\delta) + n_{NR} V_{NR}(\delta) \;\; \forall \delta < 1$$

When we analyze the lobby and non-lobby equilibria under *political constraint* we see that for the reform parameter we obtain the social surplus maximizing value in both the lobby and non-lobby game (neutrality result)[11].

Concerning the tax parameter, in the non-lobby equilibrium we have found the following equilibrium value:

$$\tilde{T}_R^{NL} = \bar{u}_{NR} - V_{NR}\left(\tilde{\delta}^{\circ}\right)$$

In the lobby equilibrium we have obtained the following transfer:

$$\tilde{T}^{\circ} - c^{\circ}_{NR}\left(\tilde{\delta}^{\circ}, \tilde{T}^{\circ}\right) = \bar{u}^{\circ}_{NR} - V^{\circ}_{NR}\left(\tilde{\delta}^{\circ}\right)$$

therefore by substitution we get:

$$\tilde{T}_R^{NL} = \tilde{T}^{\circ} - c^{\circ}_{NR}\left(\tilde{\delta}^{\circ}, \tilde{T}^{\circ}\right)$$

and since by assumption $c^{\circ}_{NR}\left(\tilde{\delta}^{\circ}, \tilde{T}^{\circ}\right) \geq 0$, it follows that:

$$\tilde{T}^{\circ} \geq \tilde{T}_R^{NL}$$

To summarize, under political constraint, in the lobby and in the non-lobby games we obtain the same reform parameter, while the transfers in the lobby game are higher than the transfers in the non-lobby game.

Note that in terms of reform parameter, the case with political constraint and the case without political constraint are quite different: under political constraint, lobby and non-lobby equilibria give exactly the same reform parameter, while in the unconstrained case the parameters can be different. This difference depends on the fact the in the unconstrained case the only way the non-rationed individuals can enter the objective function of the rationed policy maker is trough the lobby payment,

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We can see that this depends on the number of rationed versus non rationed agents:

$$\frac{n_R}{n_{NR}} > \left(\frac{V_{NR}(\delta) - V_{NR}(1)}{V_R(1) - V_R(\delta)}\right)$$

Therefore, if the number on non-rationed people is high enough, the result of the lobby game can be partial reform ($\delta^{\circ} < 1$).

[11] In fact we can see that we have the same FOCs for both the lobby and non-lobby games: $\frac{\partial L}{\partial \delta} = n_R V_R^{'}(\delta) + n_{NR} V_{NR}^{'}(\delta) = 0 \Rightarrow 0 \leq \tilde{\delta}^{\circ} = \tilde{\delta}_R^{NL} \leq 1$

therefore the lobby equilibrium is different from the non-lobby equilibrium. In the constrained case, the political constraint allows for non-rationed preferences to be considered by the policy maker, therefore - the role of the lobby payment being the same - we obtain the same reform parameter from lobbying and non-lobbying.

Finally, we can compare lobby and non-lobby equilibria in terms of utility. Let's define $\tilde{U}_i^L$ and $\tilde{G}_j^L$ the utilities of the lobby groups i and of the policy maker j in the *constrained lobby game*, $\tilde{U}_i^{NL}$ and $\tilde{G}_j^{NL}$ the utilities of the lobby groups i and of the policy maker j in the *constrained non-lobby game*. Then we can write the following:

$$\begin{aligned} \tilde{U}_{NR}^L &= \tilde{U}_{NR}^{NL} \\ \tilde{U}_R^L &\leq \tilde{U}_R^{NL} \\ \tilde{G}_R^L &\geq \tilde{G}_R^{NL} \end{aligned}$$

The non-rationed agents obtain the same payoffs in the lobby and the non-lobby game because the reform parameter and the net transfer they receive are the same. The rationed agents weakly prefer the non-lobby game because they obtain the same level of reform but they pay a higher transfer ($\tilde{T}^{\circ} \geq \tilde{T}_R^{NL}$). The policy maker weakly prefer the lobby game to the non-lobby game because he keeps the extra-transfer from the rationed to the non-rationed through the lobby payments. Therefore, in terms of reform, lobby and non-lobby game give the same result (social surplus maximizing value). The difference between the two games is just in the *distribution of surplus*: the lobby competition implies a transfer from the rationed agents to the policy maker. Therefore for the rationed lobbyist it would be better to avoid lobby competition and obtain the non-lobby equilibrium.

3 Lobbying and efficiency: a normative analysis

In the previous section we have characterized the lobby equilibrium and non-lobby equilibrium. We have derived the truthful equilibrium under alternative assumptions on the political power of the lobby groups. We have discovered that, when the losers from the reform do not have political power, compensation is not paid in the lobby equilibrium. Concerning the reform parameter, the social surplus maximizing value is chosen. On the other hand, when the losers have political power, compensation is paid in the lobby equilibrium and again the social surplus maximizing value of reform is chosen. Finally we find a *neutrality result*: when the losers have political power, the same level of reform is chosen in the lobby and non-lobby equilibrium.

Since the social surplus maximizing value is chosen in the lobby equilibrium, it is clear that this equilibrium is efficient. The social surplus maximizing value of δ can be any value between zero and one. In particular we know that, if the number of non-rationed individuals is high enough, then *partial reform,* ($\delta < 1$) is the social surplus maximizing policy. Note that the reform that gives the highest aggregate output is full liberalization, ($\delta = 1$). Furthermore, we know that there exist a pareto-improving full reform that is ($\delta = 1, T = T_{\max}$). Therefore the question arises: is the social surplus maximizing policy an efficient policy even tough it is partial reform? To answer this question we have to specify better which notion of efficiency we are using. Following Besley-Coate (1996)[12], we say that a reform is *inefficient* if, given the set of the technologically feasible utility allocations, this reform constraints the economy to the interior of the set. Or alternatively, there exists another reform that can shift the economy on the efficient frontier. In our model, if we inspect the set of the technologically feasible allocations , we will see that, given the assumptions on the preferences, the full efficient reform, ($\delta = 1, T = T_{\max}$), is not technologically feasible: given the structure of preferences, a rationed policy maker will always chose $T = T_{\min}$. Consequently, what prevents the implementation of the full efficient reform[13] is not the political game (lobby game), but the structure of the policy maker preferences. Therefore, we cannot conclude that lobbying is a source of inefficiency for the economy. All we have is an economy that remains at a lower level of output because the objective function the government doesn't allow for a better policy.

The efficiency result can be surprising if we think that lobbying is commonly viewed as a source of inefficiency. Indeed, we have chosen to model the relationship between the citizens and the policy-maker as a lobby game precisely to find an explanation of the inefficient policy making. Nevertheless, the result is less surprising if we look carefully at the way we modeled lobbying. In this theoretical framework lobbying is efficient because trough the lobby payments the individuals truthfully reveal their preferences to the policy maker so that the social surplus is maximized. Therefore, the question arises: how could we get inefficiency in this context? If we allow for not truthful payment functions, we can get inefficient solutions. This is proved in Besley-Coate (1996)[14]. On the other hand, we could get inefficiency if not all the lobbies reveal their preferences, that is, if not all the lobbies are actually lobbying. In our case we can construct an example of inefficient 1-lobby equilibrium. This happens when only the losers from the reform process are lobbying. In this case, if we compute the equilibrium policy parameter using the characterization of the lobby equilibrium à la Dixit-Grossman-Helpman, we obtain the equilibrium parameter,δ°_{NR}, from the following first order condition:

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[12] T. Besley and S. Coate, Sources of Inefficiency in a Representative Democracy: A Dinamic Analysis.

[13] Note that in the case of no political constraint, the non-lobby equilibrium is full reform and zero transfer and the lobby equilibrium is social surplus maximising reform paramenter and zero transfers. Even tough the social surplus maximizing value is less than one, moving from lobby to non-lobby is not pareto-improving. Hence we cannot say that lobby equilibrium is inefficient.

[14] T. Besley and S. Coate, Lobbying and Welfare un a Representative Democracy, 1996

$$V_R'(\delta) + n_{NR}^{\circ} V_{NR}'(\delta) = 0$$

since the social surplus maximizing value, δ°, is obtained from the following first order condition:

$$n_R V_R'(\delta) + n_{NR} V_{NR}'(\delta) = 0$$

then it is clear that :

$$\delta_{NR}^{\circ} < \delta^{\circ}$$

and therefore 1-lobby equilibrium is inefficient. In this context it becomes relevant to ask if 2-lobbies is the unique Nash equilibrium, because if this was not true, we could get inefficient policy choices as the outcome of the lobby process. We address this question in the next section.

4 Lobbying decision: solving for the equilibrium

In the previous sections we have characterized the lobby equilibrium assuming that there are two lobbies and that both are lobbying at the equilibrium. From the welfare analysis of the lobby equilibrium we have seen that the number of lobbies that is lobbying at the equilibrium is crucial for the efficiency question. Indeed we can construct an example of 1-lobby equilibrium that is inefficient. Therefore it is important to answer the following questions: *how many lobbies are actually lobbying at the equilibrium? Is 2-lobbies the unique Nash equilibrium of the lobby game?* This question is not formally analyzed in Dixit-Grossman-Helpman. They give an informal explanation of why 2-lobbies equilibrium arises, using the prisoner's dilemma argument. But in our model, because of a the asymmetric interests of the 2 lobbies[15], the existence and uniqueness of the 2-lobbies equilibrium doesn't come straightforwardly from the prisoner's dilemma argument. Since we have seen that the existence and the uniqueness of the 2-lobbies equilibrium is crucial to our welfare analysis, in what follows we investigate formally the problem.

To discover how many lobbies are actually lobbying, we have to analyze the lobbying decision of the groups in the lobby game. Let's define $s_i \in \{L, NL\}$ the set of lobbies' strategies, where:

- $L = lobbying$
- $NL = non - lobbying$

Let's define $N \in \{0, 1, 2\}$ the number of lobbying groups and let's denote N° the number of lobbying groups in the Nash equilibrium. Depending on the lobbying

[15] In Dixit-Grossman-Helpman the lobbies essentially compete for the same policy, while in our model they lobby to obtain opposite policies.

decision, we can have non-lobby ($N = 0$), 1-lobby ($N = 1$) or 2-lobby ($N = 2$) equilibrium:

	NL	L
NL	$N = 0$	$N = 1$
L	$N = 1$	$N = 2$

We want to know how many lobbies are lobbying in the Nash equilibrium of the lobby game. The results of this investigation are summarized by propositions 3-5.

Proposition 3 *Let $N \in \{0, 1, 2\}$ be the possible number lobbying groups in the lobby game, $\left(\delta^\circ, T^\circ, C^\circ_{i=R,NR}\right)$ be a truthful equilibrium with payoffs $(u^\circ_R, u^\circ_{NR})$ and N° the number of lobbying groups in the Nash Equilibrium, it doesn't exist a Nash equilibrium where $N^\circ \in \{0, 1\}$.*

The intuition behind this result is the following: in the non-lobby equilibrium the reform package chosen by the rationed policy maker is full reform without transfers. Since we know that this is the worst outcome for the non-rationed citizens, they can always propose the policy maker a new policy vector and a lobby payment that makes them better off. Therefore non-lobby is not an equilibrium. On the other hand, once non-rationed are lobbying, the policy outcome will not be the best one for the rationed citizens, hence for the same argument, rationed citizens also have an incentive to lobby and therefore 1-lobby is not an equilibrium. (proof in appendix). From the formal proof we can also see that the non-rationed individuals have a unilateral interest in lobbying, while the rationed have an interest in lobbying only in the case also non-rationed are lobbying. This happens because, when the policy maker is a rationed individual, non-lobby is the most preferred outcome for the rationed citizens[16].

Proposition 4 *Let $N \in \{0, 1, 2\}$ be the possible number of lobbying groups, $\left(\delta^\circ, T^\circ, C^\circ_{i=R,NR}\right)$ be a truthful equilibrium with payoffs $(u^\circ_R, u^\circ_{NR})$ and N° the number of lobbying groups in the Nash Equilibrium, then, if $C^\circ_R = 0$ is an equilibrium payment, it exist a Nash equilibrium where $N^\circ = 2$ and this equilibrium is unique.*

The intuition for this result is the following: to obtain 2-lobbies equilibrium, both the lobby groups must not have an incentive to deviate to non-lobby, where non-lobby means zero willingness to pay for the policy. Note that, from proposition 1 we know that the non-rationed citizens have a unilateral interest in lobbying, therefore they will

[16] Here we can see how the identity of the policy maker affects the entry decision of the lobby groups: when the policy maker is rationed, the group having unilateral interest in lobbying is the group of non-rationed (losers). Note that in this case, if 1-lobby could be an equilibrium, it would be inefficient because only losers lobby. In general, it seems that to get inefficiency we need the losers to be the only lobby group in the Nash equilibrium.

never deviate. For the rationed individuals this is not true, therefore it could happen that they find more profitable to deviate to zero willingness to pay than to play the lobby game. The advantage of deviation is that they do not pay the policy maker (but they lose in terms of policy). The advantage of lobbying is to obtain a more preferred policy against a payment. But note that in the lobby game the equilibrium payment need not to be strictly positive: even if the rationed principals have positive willingness to pay, if they have the power to lower the non-rationed's payoff as much as they want, then they can pay zero at the equilibrium. In this case, they will obtain a policy vector that is better than in the 1-lobby equilibrium paying nothing like in 1-lobby equilibrium, hence deviation is not profitable. Therefore $C_R^\circ = 0$ being an equilibrium payment is sufficient for 2-lobbies to be a Nash equilibrium (proof in appendix).

From the formal proof of this result (see appendix), we can see that under the assumption of no political power, zero payment can always be an equilibrium payment. Therefore, if there are not political constraints, 2-lobbies is the unique Nash equilibrium as it is assumed in Dixit-Grossman-Helpman. When there is a political constraint this is not always true. The political constraint implies that the rationed agent cannot lower the utility of the non-rationed as much as they want. Therefore, the sufficient condition for the existence of the equilibrium can be violated. The next proposition gives then general necessary and sufficient conditions for the existence of the 2-lobbies equilibrium.

Proposition 5 *Let $N \in \{0, 1, 2\}$ be the possible number of lobbying groups, $\left(\delta^\circ, T^\circ, C_{i=R,NR}^\circ\right)$ be a truthful equilibrium with payoffs $(u_R^\circ, u_{NR}^\circ)$ and N° the number of lobbying groups in the Nash Equilibrium, an equilibrium of the game where $N^\circ = 2$ always exists if and only if: $n_R \left[V_R(\delta^\circ) - V_R(\delta_{NR}^\circ)\right] + n_{NR} \left[V_{NR}(\delta^\circ) - V_{NR}(\delta_{NR}^\circ)\right] > 0$*

Intuition: from proposition 2 we know that if zero payment is an equilibrium payment of the lobby game, then 2-lobbies is the unique Nash equilibrium. For zero payment to be an equilibrium payment, the rationed agent must have the power to lower the non-rationed payoff, u_{NR}°, as much as they want. But when the non-rationed have political power this is not possible. Therefore for an equilibrium to always exists (where "always" means even if $C_R^\circ = 0$ is not an equilibrium payment[17]), we need that the payoff from lobbying is higher then the payoff from deviation for every strictly positive lobby payment and this happens to be true when the social surplus under 2-lobbies equilibrium is higher than the social surplus under 1-lobby equilibrium.(proof in appendix).

Corollary 1 : *2-lobbies is the unique Nash equilibrium and it is efficient.*

[17] From proposition 1 we know that $C_R^\circ = 0$ is sufficient for the existence of a NE, from proposition 3 we can see that $C_R^\circ = 0$ is not necessary for the existence of a Nash equilibrium.

Proof.

• *existence and efficiency*: from proposition 3 we know that when the social surplus under 2-lobbies equilibrium is higher than the social surplus under 1-lobby equilibrium, then the equilibrium with $N^{\circ} = 2$ exists. Since the equilibrium policy outcome, $(\delta^{\circ}, T^{\circ})$, of the 2-lobbies game is the social surplus maximizing value, the necessary and sufficient condition for the existence of the equilibrium are always satisfied. From the social surplus maximizing property we also deduce that the equilibrium is efficient.

• *uniqueness*: from proposition 1 we know that $N^{\circ} \in \{0, 1\}$ is not a Nash equilibrium, hence $N^{\circ} = 2$ is the unique Nash equilibrium.

To summarize, the result of the investigation on the lobbying decision is that 2-lobbies is the unique Nash equilibrium and it is efficient. In this sense, we obtain the same results of Dixit-Grossman-Helpman, even tough in our model we added a political constraint and asymmetric lobbies' interests. The introduction of the political constraint doesn't have any implication for the existence and uniqueness of the 2-lobbies equilibrium. The effect of the political constraint is just in terms of equilibrium pay-off: depending on how binding is the constraint, we can exclude some equilibria. For example, from the formal proof of proposition 4, we have seen that because of the political constraint it is not always true that zero payment for the rationed,$C_R^{\circ} = 0$, is an equilibrium payment. In other terms, when the non-rationed individuals have political power, the rationed individuals are not necessarily able to shift the entire cost of lobbying on the non-rationed. Hence the political constraint has an effect on the sharing of lobby cost or alternatively on the final payoffs of the game.

Concerning the asymmetric interests of the lobby groups, the main consequence of this assumption is that the two groups do not both have a unilateral interest in lobbying: the non-rationed individuals have interest in lobbying whatever the other group is doing, the rationed have an interest in lobbying only if non-rationed too lobby. Nevertheless, adding this asymmetry to the Dixit-Grossman-Helpman framework is not sufficient to change the number of groups that are lobbying at the Nash equilibrium. Even tough only non-rationed have a unilateral interest in lobbying, at the Nash equilibrium both rationed and non-rationed are lobbying. Since we know from the welfare analysis that an inefficient equilibrium can arise if not all the lobbies are lobbying, we think that a further investigation on the condition that can support inefficient equilibria is necessary. In this sense we think that we probably need to change the timing of the game and the structure of the lobby costs. For example,it could be appropriate to consider a different timing of the game where the group that has a unilateral interest in lobbying is the first mover. We also think that the cost setting could be made more rich adding some fixed component (for example, a cost of entry). More in general, our opinion is that the efficiency result of a model à la Dixit-Grossman-Helpman is driven by a sort of perfect competition in lobbying that doesn't necessarily capture in a realistic way the lobby process.

Therefore, we think that it could be worth investigating the lobby phenomenon in a different theoretical framework where the various groups of individuals, because of their preferences (some groups have stronger interests than other in lobbying) and of the structure of the lobby costs, are not necessarily *all* lobbying for policy concessions.

5 Concluding Remarks

In this paper we undertake a positive and normative analysis of public decision making, considering the case of a reform (price liberalization) which creates gainer and losers among the citizens. This analysis was motivated by the massive reform process of the Eastern European countries in transition from central planning to the market regime. The transition process is a major phenomenon where economic, political and sociological factors are strictly related. We are mainly concerned with the economic side of the process and structural reforms (such as price liberalization, privatization and restructuring) are an important part of it. The central question behind this work is the following: *are efficient economic reforms undertaken?* Although the notion of efficiency itself can be controversial, using the concept of efficient policy in the sense of Besley-Coate (1996), we tackle the problem in a theoretical framework where policy decisions are the outcome of a political game between the policy maker and the citizens. The policy maker has to choose a policy parameter which creates gainers and losers among the citizens. Non-distortive taxation can be used to compensate losers and the gain from full reform are high enough to make everybody better off with respect to the *status quo*. Following Dixit-Grossman-Helpman (1996), we model the political game using the menu auction approach: the citizens, organized in lobbies, are the bidders and the policy maker is the auctioneer. Lobbies offer a policy and a payment according to a truthful payment function and the government, given all the offers, takes the policy decisions. Because of truthful revelation of the individual preferences, the government implements the social surplus maximizing policy. Consequently, the equilibrium of the lobby game is efficient, even though compensating transfers are not always paid. We also constructed an example of inefficient policy, which arises when not all the lobbies participate in the auction. However, we find that this is not a Nash equilibrium. The unique Nash equilibrium of the model is the equilibrium where *all* the lobbies are actually lobbying. Therefore, in this theoretical framework there is no space for inefficient policy making. Nevertheless, we think that the problem of the *government failure* in the policy making process deserves further investigation. Therefore, to move in the right direction, a substantial extension of this model is necessary.

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APPENDIX

Truthful Equilibrium with no political constraint

(a)$_{\text{R}}$ max problem for the policy-maker (*rationed*)

The first condition characterizing the lobby equilibrium says that the equilibrium policy chosen by the policy maker must maximize his objective function. Let $(\delta^\circ, T^\circ, C_{NR}(\delta^\circ, T^\circ, u_{NR}^\circ), C_R(\delta^\circ, T^\circ, u_R^\circ))$ be a truthful equilibrium where $(u_R^\circ, u_{NR}^\circ)$ are the payoffs of the principals, then from the condition (a) of the RESULT 2-DGH, the policy vector, (δ°, T°), must satisfy the following condition:

$$(\delta^\circ, T^\circ) = \arg\max_{\delta,T} V_R(\delta) - \alpha T + n_{NR} C_{NR}[\delta, T, u_{NR}^\circ] + (n_R - 1) C_R[\delta, T, u_R^\circ]$$

replacing for the expression of $C_R[\delta, T, u_R^\circ]$ and $C_{NR}[\delta, T, u_{NR}^\circ]$, the objective function becomes:

$G_R[\delta, T] = n_R V_R(\delta) + n_{NR} V_{NR}(\delta) - K$

$K = n_{NR} u_{NR}^\circ + (n_R - 1) u_R^\circ$

that is just the social surplus less a constant. Therefore the maximization problem of the policy maker gives the social surplus maximizing value of δ[18]:

$\max_{\delta} n_R V_R(\delta) + n_{NR} V_{NR}(\delta)$

FOCs(j):

1) Interior solution for δ :

$\delta^\circ : n_R V_R'(\delta) + n_{NR} V'(\delta) = 0 \Rightarrow 0 \leq \delta^\circ \leq 1$

Proof. :

$n_R V_R'(\delta) > 0 \ \forall \delta \geq 0$

$n_{NR} V'(\delta) < 0 \ \forall \delta \geq 0$

therefore

$n_R V_R'(\delta) + n_{NR} V'(\delta) = 0 \ \forall \delta \geq 0$

■

(b) participation constraint that principals must satisfy

Concerning the second part of the proposition (condition (b)), it says that each lobby group must offer the policy maker a policy vector,(δ°, T°), and a payment, C_i°, that satisfy the participation constraint. Therefore we

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[18] If the policy- maker is non-rationed we have:

(a)$_{\text{NR}}$ max problem for the policy-maker (*non-rationed*)

$(\delta^\circ, T^\circ) = \arg\max_{\delta,T} V_{NR}(\delta) + T + (n_{NR} - 1) C_{NR}[\delta, T, u_{NR}^\circ] + n_R C_R[\delta, T, u_R^\circ]$

note that the maximisation problem doesn't depend on the identity of the policy maker. Once again we get the social surplus maximizing value:

$\max_{\delta} n_{NR} V_{NR}(\delta) + n_R V_R(\delta) - k'$

FOCs(j):

2) Interior solution for $\delta : 0 \leq \delta^\circ \leq 1$.

write the participation constraint for both the rationed and non-rationed principals.

- *Rationed principal vs rationed policy-maker:*

$(\mathrm{b})_{\mathrm{R}}$ $n_{NR}V_{NR}(\delta^{\circ}) + n_R V_R(\delta^{\circ}) - n_{NR}u^{\circ}_{NR} - (n_R - 1)u^{\circ}_R = \max_{\delta,T} V_R(\delta) - \alpha T + n_{NR}C_{NR}[\delta, T, u^{\circ}_{NR}]$

The LHS of the condition $(\mathrm{b})_{\mathrm{R}}$ is the utility the rationed principal offers the policy maker and the RHS is the utility the policy maker can obtain from his outside option. To distinguish the case where the two principals are lobbying from the case where only one principal is lobbying we introduce the following notation: we denote $(\delta^{\circ}, T^{\circ})$ the equilibrium parameters when the two principals are lobbying and $(\delta^{\circ}_i, T^{\circ}_i)$the equilibrium parameters when only group i is lobbying, where $i = \{R, NR\}$.

Consider now the participation constraint $(\mathrm{b})_{\mathrm{R}}$ .To solve the maximization problem of the RHS of the $(\mathrm{b})_{\mathrm{R}}$ equation, let's consider the FOCs:

$\max_{\delta,T} V_R(\delta) - \alpha T + n_{NR}[V_{NR}(\delta) + T - u^{\circ}_{NR}] \equiv V_R(\delta) + n_{NR}V_{NR}(\delta) + (n_R - 1)\alpha T - n_{NR}u^{\circ}_{NR}$

FOCs:

1)$\delta : V'_R(\delta) + n^{\circ}_{NR}V'(\delta) = 0 \Longrightarrow \delta^{\circ}_{NR} < \delta^{\circ}$

2) corner solution for T $\Longrightarrow T^{\circ}_{NR} = T^{\circ}_{\max}$

Therefore, we can write the participation constraint as follows:

$(\mathrm{b})^{\circ}_{\mathrm{R}}$ $n^{\circ}_R V^{\circ}_R(\delta^{\circ}) - V^{\circ}_R(\delta^{\circ}_{NR}) + n^{\circ}_{NR}[V^{\circ}_{NR}(\delta^{\circ}) - V^{\circ}_{NR}(\delta^{\circ}_{NR})] - (n^{\circ}_R - 1)\alpha T_{\max} = (n_R - 1)\, u^{\circ}_R$

- *Non rationed principal vs rationed policy-maker:*

Again we write the participation constraint for the non-rationed principal as the equality between the utility the non-rationed offers the policy maker and the utility of the policy maker's outside option:

$(\mathrm{b})^{\circ}_{\mathrm{NR}}$ $n^{\circ}_{NR}V^{\circ}_{NR}(\delta^{\circ}) + n^{\circ}_R V^{\circ}_R(\delta^{\circ}) - n^{\circ}_{NR}u^{\circ}_{NR} - (n^{\circ}_R - 1)u^{\circ}_R = \max_{\delta,T} V_R(\delta) - \alpha T + (n_R - 1)C_R[\delta, T, u^{\circ}_R]$

Solving again the RHS, we can see that FOCs are not verified and we get corner solutions for both variables:

$\max_{\delta,T} n_R V_R(\delta) - n_{NR}T - (n_R - 1)u^{\circ}_R$

1) corner solution for T $\Longrightarrow T^{\circ}_R = 0$

2)corner solution for $\delta \Rightarrow \delta^{\circ}_R = 1$

then we can write the participation constraint $(\mathrm{b})_{\mathrm{NR}}$ as follows:

$(\mathrm{b})_{\mathrm{NR}}$ $n_R[V_R(\delta^{\circ}) - V_R(\delta^{\circ}_R)] + n_{NR}V_{NR}(\delta^{\circ}) + n_{NR}T_{\min} = n_{NR}u^{\circ}_{NR}$

To summarize, the lobby equilibrium characterized by the RESULT 2-DGH shows a reform parameter, δ°, that can assume any value between zero and one satisfying the first order condition of the policy maker maximization problem $(n_R V'_R(\delta) +$

$n_{NR}V'(\delta) = 0$). Concerning the tax parameter, we can see that it doesn't appear in the objective function of the policy makers. Finally the equilibrium payoffs of the game, u°_R and u°_{NR}, have been derived from the equation $(\text{b})_{\text{NR}}$ and $(\text{b})_{\text{R}}$:

$(\text{b})_{\text{R}}$ $(n_R - 1) V_R(\delta^{\circ}) + [V_R(\delta^{\circ}) - V_R(\delta^{\circ}_{NR})] + n_{NR}[V_{NR}(\delta^{\circ}) - V_{NR}(\delta^{\circ}_{NR})] - (n_R - 1)\alpha T_{\max} = (n_R - 1) u^{\circ}_R$

$(\text{b})_{\text{NR}}$ $n_R[V_R(\delta^{\circ}) - V_R(\delta^{\circ}_R)] + n_{NR}V_{NR}(\delta^{\circ}) + n_{NR}T_{\min} = n_{NR}u^{\circ}_{NR}$ ■

Binding political constraint

Proof. :

Let's assume first that the constraint is binding. Then we prove that if it is not, the objective function is not maximized.

Consider our maximization problem:

$\max_{\delta} V_R(\delta) - \alpha T$

st (1) $V_{NR}(\delta) + T \geq \bar{u}_{NR}$

Hp1 (*binding constraint*): $T = \bar{u}_{NR} - V_{NR}(\delta)$

Under Hp1 we can rewrite the maximization problem as follows:

$\max_{\delta} V_R(\delta) - \alpha \left(\bar{u}_{NR} - V_{NR}(\delta)\right)$

Imposing the first order condition we obtain:

$n_R V'_R(\delta) + n_{NR}V'(\delta) = 0 \Rightarrow 0 \leq \tilde{\delta}^{NL}_R \leq 1$

therefore, replacing $\tilde{\delta}^{NL}_R$ in the binding constraint we obtain:

$\tilde{T}^{NL}_R = \bar{u}_{NR} - V_{NR}\left(\tilde{\delta}^{NL}_R\right)$

Suppose now that the constraint is not binding. This means that we have to find $\widehat{T}^{\circ} > \tilde{T}^{NL}_R$ that solves the maximization problem.

To do this let's define:

(2) $\widehat{T} = \bar{u}_R - V_{NR}(\delta) + k$, $k \in \Re^+$

note that solving the maximizations problem under the constraint (2) is equivalent to solving it under the constraint (1) not binding:

$\widehat{T} = \bar{u}_R - V_{NR}(\delta) + k > \bar{u}_{NR} - V_{NR}(\delta)$

therefore we write the maximization problem as:

$\max_{\delta} V_R(\delta) - \alpha \left(\bar{u}_{NR} - V_{NR}(\delta) + k\right)$

solving it we have:

FOC: $n_R V'_R(\delta) + n_{NR}V'(\delta) = 0 \Rightarrow 0 \leq \tilde{\delta}^{NL}_R \leq 1$

therefore the transfer in equilibrium is:

$\widehat{T}^{\circ} = \bar{u}_R - V_{NR}\left(\tilde{\delta}^{NL}_R\right) + k$

If $\widehat{T}^{\circ}$ maximizes the objective function, comparing $\widehat{T}^{\circ}$ and $\tilde{T}^{NL}_R$ we must have:

$V_R\left(\overset{\sim NL}{\delta_R}\right)-\alpha\,\widehat{T^{\circ}}> V_R\left(\overset{\sim NL}{\delta_R}\right)-\alpha\,\overset{\sim NL}{T_R}\Leftrightarrow\widehat{T^{\circ}}<\overset{\sim NL}{T_R}$ that is a contradiction.
■

Proof. of proposition 3

• **Non-lobby equilibrium**

We start with the non-lobby case and we prove that this is not an equilibrium because there is at least one group that has an interest to deviate and offer a positive payment.

Consider the payoff of the non-rationed group in the non-lobby equilibrium. Let's define $\Delta\varepsilon$ and $\Delta\tau$ as two marginal variations from the non-lobby equilibrium policy parameters. We can then see that non-lobby is not an equilibrium because the following inequality holds:

$V_{NR}^{\circ}\left(\delta_R^{NL}\right)+T_R^{NL}\leq V_{NR}^{\circ}\left(\delta_R^{NL}+\Delta\varepsilon\right)+\left(T_R^{NL}+\Delta\tau\right)-C_{NR}^{\circ}\left(\delta+\Delta\varepsilon,T+\Delta\tau,u_{NR}^{\circ}\right)$

$\Delta\varepsilon\to 0\quad\Delta\tau\to 0$

the LHS is the payoff under non-lobby equilibrium and the RHS is the payoff from a marginal deviation from the non-lobby equilibrium.

Let's define:

$\Delta U_{NR}=V_{NR}^{\circ}\left(\delta_R^{NL}+\Delta\varepsilon\right)+\left(T_R^{NL}+\Delta\tau\right)-V_{NR}^{\circ}\left(\delta_R^{NL}\right)-T_R^{NL}$

$\Delta C_{NR}=C_{NR}^{\circ}\left(\delta+\Delta\varepsilon,T+\Delta\tau,u_{NR}^{\circ}\right)-0$

Since (δ_R^{NL},T_R^{NL}) is not the most preferred outcome for the NR agents, we can find a marginal variation of δ and T that can increase the NR utility, $\Delta U_{NR}>0$,and we can always find a positive payment inferior to the marginal utility gain, $\Delta C_{NR}<\Delta U_{NR}$, that makes profitable to lobby. Therefore we have proved that there is at least one group that has a unilateral interest in deviating from the non-lobby equilibrium. We can also prove that this is the only group that has unilateral interest in deviating from the non-lobby equilibrium. To see this, consider the rationed group. It is immediate to verify that, since the non-lobby equilibrium outcome, (δ_R^{NL},T_R^{NL}) is identical to their most preferred outcome, $(\delta_R^{\circ},T_R^{\circ})$:

$$\delta_R^{NL}=\delta_R^{\circ}\ and\ T_R^{NL}=T_R^{\circ}$$

any marginal variation from that equilibrium decreases their utility[19]. Therefore it is not possible to find a positive lobby payment that can make lobbying profitable.

• **1-lobby equilibrium**

Since we know that NR have a unilateral interest to lobby, while R have not the same interest, we have to control if 1-lobby can be an equilibrium. This is true if R do not have any interest to join the lobby game. Again, since the two policy parameters chosen when NR is the only lobby group, $(\delta_{NR}^{\circ},T_{NR}^{\circ})$ are different from R's most preferred policy, $(\delta_R^{\circ},T_R^{\circ})$, we can always find a marginal variation of the

[19] write FOCs from wich we get the two outcomes.

parameters that increases R's utility and a positive payment, inferior to the marginal utility gain, such that joining lobby is profitable for R. Therefore 1-lobby is not a Nash equilibrium.

■

Proof. of proposition 4

- **2-lobbies equilibrium**

* *Uniqueness*: To prove the uniqueness consider that, from proposition 1, non-lobby and 1-lobby are not Nash Equilibria, therefore if 2-lobbies is a Nash equilibrium, it will be unique.

* *Existence*: If the Nash equilibrium where $N^{\circ} = 2$ exists, NR and R must not have an interest to deviate to zero willingness to pay for the policy. Since for i=NR we have already proved that there is a unilateral interest in lobbying, we know that for this group there is no interest to deviate to non-lobby. For the other group it is not obvious that there is not interest to deviate to zero. To check if R do not have interest to deviate to non-lobby we compare utility of deviation, U_R^D, and utility of lobbying,U_R^L :

$$\begin{aligned} U_R^D(\delta, T) &= V_R(\delta) - \alpha T \\ U_R^L[\delta, T, C_R(\delta, T, u_R)] &= V_R(\delta) - \alpha T - C_R(\delta, T, u_R) \end{aligned}$$

note that:

$$U_R^L = U_R^D(\delta, T) - C_R(\delta, T, u_R)$$

therefore:

$$U_R^D(\delta_{NR}^{\circ}, T_{NR}^{\circ}) = U_R^L[\delta_{NR}^{\circ}, T_{NR}^{\circ}, 0]$$

where $(\delta_{NR}^{\circ}, T_{NR}^{\circ})$ are the equilibrium policy parameters when only the group NR is lobbying.

The difference between lobby and deviation is that in the first case individuals have positive willingness to pay while in the second they have zero willingness to pay. Note that the fact that in the lobby equilibrium R have a positive willingness to pay doesn't mean that the equilibrium payment will be necessarily positive. In fact, from the definition of the payment function we know that we can always find u_R° such that $C_R^{\circ}(\delta^{\circ}, T^{\circ}, u_R^{\circ}) = 0$.

Furthermore, from condition (c) of Result 1 in Dixit-Grossman-Helpman we know that:

$$\forall i = R, NR \Longrightarrow [\delta^{\circ}, T^{\circ}, c^{\circ}] = \arg\max_{\delta,T,c} U_i^L [\delta, T, c] \tag{c}$$

therefore, we can in particular say that:

$$(\delta^{\circ}, T^{\circ}, 0) = \arg\max_{\delta,T,c} U_R^L [\delta, T, c]$$

which means that $U_R^L [\delta^{\circ}, T^{\circ}, 0]$ is the maximum level of utility R can attain under zero payment. In particular this means that:

$$U_R^L [\delta^{\circ}, T^{\circ}, 0] > U_R^L [\delta_{NR}^{\circ}, T_{NR}^{\circ}, 0] = U_R^D (\delta_{NR}^{\circ}, T_{NR}^{\circ})$$

therefore R do not have an incentive to deviate to non-lobby. In other words, when zero payment is a possible outcome of the lobby game, the condition (c) in RESULT 1-DGH insures that lobbying gives a higher utility than deviation ($c_R^{\circ} = 0$ is sufficient for no deviation, hence it the sufficient for the existence of the equilibrium).

We have now to check if $c_R^{\circ} = 0$ doesn't violate the participation constraint. If $c_R^{\circ} = 0$ satisfies the *participation constraint* of RESULT 1-DGH, we must have:

$$G_R (\delta^{\circ}, T^{\circ}, c_{NR}^{\circ}, 0) = \max_{\delta', T'} G_R \left(\delta', T', C_{NR} \left(\delta', T'\right), 0\right) \tag{d1}$$

where G_R is the utility of the policy maker. The participation constraint in equation d1 says that, in the relationship between the rationed agent and the policy maker, the rationed agent must provide the policy maker at least the level of utility that the policy maker can obtain from his outside option, that is the utility the policy maker obtains playing his best response given that the non-rationed offer $C_{NR} \left(\delta', T'\right)$ and the rationed offers nothing (LHS of equation d1).

Note that if we write the *participation constraint* for every positive payment we have:

$$G_R \left(\delta^{\circ}, T^{\circ}, \hat{c}_{NR}^{\circ}, \hat{c}_R^{\circ}\right) = \max_{\delta', T'} G_R \left(\delta', T', C_{NR} \left(\delta', T'\right), 0\right) \tag{d2}$$

from (d1)and (d2) it follows that:

$$G_R (\delta^{\circ}, T^{\circ}, c_{NR}^{\circ}, 0) = G_R \left(\delta^{\circ}, T^{\circ}, \hat{c}_{NR}^{\circ}, \hat{c}_R^{\circ}\right) \tag{d3}$$

which implies:

$$c_{NR}^{\circ} > \hat{c}_{NR}^{\circ}$$

more precisely, G_R is defined as follows:

$$G_R [\delta, T, C_{NR} [\delta, T], C_R [\delta, T]] = V_R (\delta) - \alpha T + n_{NR} C_{NR} [\delta, T] + (n_R - 1) C_R [\delta, T]$$

using the definition of G_R and replacing for the equilibrium values in the equality of equation d3 we obtain:

$$c_{NR}^{\circ} = \overset{\wedge\circ}{c}_{NR} + \frac{(n_R - 1)}{n_{NR}} \overset{\wedge\circ}{c}_{R} \tag{8}$$

which means that when $c_R^{\circ} = 0$, the non-rationed agent is paying alone the total "cost" of lobbying for $(\delta^{\circ}, T^{\circ})$, while in the case of $\left(\overset{\wedge\circ}{c}_{NR}, \overset{\wedge\circ}{c}_{R}\right)$ both positive, this "cost" is shared between R and NR.

It is therefore clear that, if you can higher the payment of the non-rationed individual, c_{NR}°, as much as you want, the equation 8 is always satisfied.

Note that, since by definition:

$$c_{NR}^{\circ} = V_{NR}(\delta^{\circ}) + T^{\circ} - u_{NR}^{\circ}$$

it follows that, if there is not constraint on u_{NR}°, you can lower it as much as you want, which means that you can higher c_{NR}° as much as you want (or, equivalently, you can higher u_R° as much as you want). Under the assumption of no political power we do not have any constraint on u_{NR}°, we conclude that equation 8 is always satisfied and therefore zero payment, satisfying the participation constraint, can be an equilibrium payment of the lobby game. Under the assumption of no political power this is not necessarily true. When rationed agent have political power we have the following political constraint:

$$U_{NR}(\delta, T, C_{NR}(\delta, T, u_{NR})) = V_{NR}(\delta) + T - C_{NR}(\delta, T, u_{NR}) \geqslant \bar{u}_{NR}$$

it says that the utility of the rationed agent, U_{NR} , cannot be inferior to a given reservation utility $\bar{u}_{NR}$.

Let $\left(\overset{\sim\circ}{\delta}, \overset{\sim\circ}{T}, \overset{\sim\circ}{c}_{NR}, \overset{\sim\circ}{c}_{R}\right)$ be a truthful equilibrium of the constrained lobby game where $(u_{NR}^{\circ}, u_R^{\circ})$ are the payoffs and $N^{\circ} = 2$. Again we want to prove that this is the Nash equilibrium.

The first part of the previous proof applies again to the lobbies equilibrium under political constraint: if zero payment is a possible outcome of the lobby game, deviation is not profitable.

Now we check that zero payment doesn't violate the participation constraint. We can see that, in the constrained game, u_{NR}° is no more a free parameter. From the political constraint evaluated at the equilibrium point we have:

$$\overset{\sim\circ}{u}_{NR} = V_{NR}\left(\overset{\sim\circ}{\delta}\right) + \overset{\sim\circ}{T} - \overset{\sim\circ}{c}_{NR} \geqslant \bar{u}_{NR}$$

In terms of payment this means:

$$\overset{\sim\circ}{c}_{NR} \leqslant V_{NR}\left(\overset{\sim\circ}{\delta}\right) + \overset{\sim\circ}{T} - \bar{u}_{NR}$$

consider now that the for $\overset{\sim\circ}{c}_{R} = 0$ to satisfy the participation constraint it must be that:

$$\overset{\sim\circ}{c}_{NR} = \overset{\wedge\circ}{c}_{NR} + \frac{(n_R - 1)}{n_{NR}} \overset{\wedge\circ}{c}_{R}$$

then :

• if $\overset{\wedge\circ}{c}_{NR} + \frac{(n_R-1)}{n_{NR}} \overset{\wedge\circ}{c}_{R} \leqslant V_{NR}\left(\overset{\sim\circ}{\delta}\right) + \overset{\sim\circ}{T} - \bar{u}_{NR} \Longrightarrow \overset{\sim\circ}{c}_{R} = 0$ satisfies the participation constraint

• if $\overset{\wedge\circ}{c}_{NR} + \frac{(n_R-1)}{n_{NR}} \overset{\wedge\circ}{c}_{R} > V_{NR}\left(\overset{\sim\circ}{\delta}\right) + \overset{\sim\circ}{T} - \bar{u}_{NR} \Longrightarrow \overset{\sim\circ}{c}_{R} = 0$ violates the participation constraint ■

Proof. of proposition 5

Let's consider the case of $N = 2$ and let's prove proposition 5 in this case. For $N^\circ = 2$ it must be that for both lobby groups lobbying is better then deviation. For the non-rationed individuals we already know that lobbying dominates non-lobbying (from proposition 1 we know that non-lobby is not an equilibrium because NR has a unilateral interest in lobbying). For the rationed individuals, let's compare the *payoff from lobbying* and the *payoff from deviation* . We say that lobbying is better then deviation iff the following is true:

$$V_R\left(\overset{\sim\circ}{\delta}\right) - \alpha \overset{\sim\circ}{T} - C_R\left(\overset{\sim\circ}{\delta}, \overset{\sim\circ}{T}, \overset{\sim\circ}{u}_R\right) > V_R\left(\delta^\circ_{NR}\right) - T^\circ_{NR} \tag{9}$$

where $\left(\overset{\sim\circ}{\delta}, \overset{\sim\circ}{T}\right)$ are the policy parameters chosen under lobbying and $(\delta^\circ_{NR}, T^\circ_{NR})$are the policy parameters chosen under deviation, that is when only NR are lobbying. With some algebra, we can see that 9 is equivalent to the inequality that we use in the statement of proposition 3. Let's prove first that inequality of proposition 3, $n_R\left[V_R\left(\delta^\circ\right) - V_R\left(\delta^\circ_{NR}\right)\right] + n_{NR}\left[V_{NR}\left(\delta^\circ\right) - V_{NR}\left(\delta^\circ_{NR}\right)\right] > 0$, is equivalent to 9, then we prove that this is necessary and sufficient for a Nash equilibrium to always exist.

Consider condition 9. By definition $C_R\left(\overset{\sim\circ}{\delta}, \overset{\sim\circ}{T}, \overset{\sim\circ}{u}_R\right)$ is:

$$C_R\left[\overset{\sim\circ}{\delta}, \overset{\sim\circ}{T}, \overset{\sim\circ}{u}_R\right] = V_R\left(\overset{\sim\circ}{\delta}\right) - \alpha \overset{\sim\circ}{T} - \overset{\sim\circ}{u}_R \tag{10}$$

replacing (10) in (9) we obtain the following condition:

$$V_R\left(\delta^\circ_{NR}\right) - \alpha T^\circ_{NR} < \overset{\sim\circ}{u}_R \tag{11}$$

Note that since $T_{NR}^{\circ} = T_{\max}$ and by assumption $T_{\max} : U_R[\delta, T_{\max}, C_R(\delta, T)] \geq \bar{u}_R \geq u_R^{SQ}$, the payoff from the 1-lobby game cannot be inferior to the status quo utility. Indeed since the non-rationed individuals are optimizing they will choose the $T_{\max}$ such that $U_R[\delta, T_{\max}, C_R(\delta, T)] = \bar{u}_R \geq u_R^{SQ}$. Therefore, 2-lobbies is better then 1-lobby iff:

$$\tilde{u}_R^{\circ} > \bar{u}_R \geq u_R^{SQ}$$

The equilibrium value of $\tilde{u}_R^{\circ}$ obtained from the model (equation $(b)_{\mathrm{R}}$) is:
$n_R V_R(\delta^{\circ}) - V_R(\delta_{NR}^{\circ}) + n_{NR}[V_{NR}(\delta^{\circ}) - V_{NR}(\delta_{NR}^{\circ})] - (n_R - 1)\alpha T_{\max} = (n_R - 1)\, u_R^{\circ}$

where δ_{NR}° is the NR's most preferred reform parameter.
Therefore, replacing for the expression of $\tilde{u}_R^{\circ}$ in the inequality 11 we obtain:

$$n_R[V_R(\delta^{\circ}) - V_R(\delta_{NR}^{\circ})] + n_{NR}[V_{NR}(\delta^{\circ}) - V_{NR}(\delta_{NR}^{\circ})] > 0 \tag{12}$$

where $[V_R(\delta^{\circ}) - V_R(\delta_{NR}^{\circ})] > 0$ is the gain (in terms of reform parameter) from 2-lobbies equilibrium for the rationed individuals and $[V_{NR}(\delta^{\circ}) - V_{NR}(\delta_{NR}^{\circ})] < 0$ is the loss for the non-rationed. Therefore, 2-lobbies is better then 1-lobby provided that the total gain to the rationed individuals is higher then the total loss to the non-rationed ones or, equivalently, if the social surplus in the 2-lobbies equilibrium is higher then the social surplus in the 1-lobby equilibrium.

• **Sufficiency**: for an equilibrium of the 2-lobby game to exist, we have to prove that the R's payoff from lobbying is higher then the payoff from deviation. Under condition 12, this is true $\forall\ C_R^{\circ} \geq 0$, then condition 12 is sufficient for the existence of the equilibrium.

• **Necessity**: consider R's payoff: if $C_R^{\circ} = 0$ is an equilibrium payment, then the payoff from lobbying is higher then the payoff from deviation (12 holds) and therefore an equilibrium exists. If $C_R^{\circ} = 0$ is not an equilibrium payment and 12 holds, then the payoff from lobbying is higher than the payoff from deviation and therefore an equilibrium exists. If $C_R^{\circ} = 0$ is not equilibrium payment and 12 is not satisfied, an equilibrium doesn't exist. Hence, since for an equilibrium to always exist, the R's payoff from lobbying must be higher than the payoff from deviation $\forall\ C_R^{\circ} \geq 0$, then condition 12 is necessary. ■