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Monetary Policy with Internal Central Bank Forecasting: A Case of Heterogenous Information*

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Abstract

Honkapohja and Mitra (2003) have analyzed the desirability of optimal and *ad hoc* interest rules in monetary policy when the forecasts of the private sector and the central bank are heterogenous but information is symmetric. Here we analyze the case of asymmetric information in which one party does not observe all observable shocks that the other party sees.

Key words: Adaptive learning, stability, heterogeneity, asymmetric information, monetary policy.

JEL classification: E52, E31, D84.

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1 Introduction

Recent literature on the conduct of monetary policy has usually assumed that the policy instrument is the nominal interest rate in the economy, see e.g. (Clarida, Gali, and Gertler 1999) for a survey. Optimal policies and *ad hoc* instrument rules have been examined for also determinacy of equilibria and stability of equilibria under learning, see (Evans and Honkapohja 2003a) for a review of recent research in this area. The basic message of the literature is that conditions for determinacy and stability of the equilibrium under adaptive learning can be expressed as constraints between the parameters of the policy rule and structural parameters of the economy. Good monetary policy should respect these constraints so as to avoid fluctuations that would otherwise arise.

The literature has examined a variety of policy rules for the determinacy and learnability constraints. An important case are the rules in which the instrument of monetary policy responds to forecasts of future endogenous variables. This is important since in practice there can be lags in the effects of policy and, moreover, responses to lagged data can by itself contribute to fluctuations in the economy.

A difficulty with interest rate rules that respond to private forecasts of inflation and economic activity is that the relevant forecasts are in fact private forecasts since they presumably influence private economic behavior and there can be errors in measuring private expectations of inflation and economic activity. If there are large measurement errors, then internal forecasts by the central bank can be considered a proxy for private forecasts. (Honkapohja and Mitra 2003) have examined the learnability of equilibria under interest rate rules that depend on central bank internal forecasts. Their analysis focuses on the basic case of symmetric information, i.e. both central bank and private sector forecasts are based on the same data on macroeconomic variables.

The assumption of symmetric information is a natural starting point, but obviously differences in the information sets can exist and they raise further issues for viability of monetary policy. In this paper we examine some issues of asymmetric information in forecasting between the private sector and the central bank. We take up a simple but plausible case of asymmetric information. One agent, say the private sector, has superior (full) information, as it observes both of the two shocks, while the other agent sees only one shock. An alternative assumption is that the central bank has full and the private sector limited information and we also present the results for this case.[1] We derive the effects of the postulated information asymmetry on the learnability constraints that good monetary policy should respect.

2 Analytical Framework

We employ a standard log-linearized model of monopolistic competition with price stickiness as outlined e.g. in (Clarida, Gali, and Gertler 1999). The structural model consists of an IS curve and a New Phillips curve:

$$\begin{aligned} z_t &= -\varphi(i_t - \hat{E}_t^P \pi_{t+1}) + \hat{E}_t^P z_{t+1} + g_t, && (1) \\ \pi_t &= \lambda z_t + \beta \hat{E}_t^P \pi_{t+1} + u_t, && (2) \end{aligned}$$

where z_t is the "output gap", π_t is the inflation rate, and i_t is the nominal interest rate. $\hat{E}_t^P \pi_{t+1}$ and $\hat{E}_t^P z_{t+1}$ denote *private sector* expectations of inflation and output gap next period. The same notation without the "ˆ" and superscript P denotes RE of the private sector. The parameters in (1) and (2) are positive. $0 < \beta < 1$ is the discount rate.

g_t and u_t denote observable shocks following AR(1) processes:

$$\begin{pmatrix} g_t \\ u_t \end{pmatrix} = F \begin{pmatrix} g_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \hat{g}_t \\ \hat{u}_t \end{pmatrix}, \; F = \begin{pmatrix} \mu & 0 \\ 0 & \rho \end{pmatrix}, \tag{3}$$

where $0 < \mu < 1, 0 < \rho < 1$ and $\hat{g}_t \sim iid(0, \sigma_g^2), \hat{u}_t \sim iid(0, \sigma_u^2)$. g_t and u_t are the demand and "cost push" shocks, respectively.

We supplement equations (1) and (2) with a rule for the nominal interest rate i_t in which the interest rate is adjusted in accordance with the central bank expectations of output gap and inflation. Then

$$i_t = \chi_0 + \chi_\pi \hat{E}_t^{CB} \pi_{t+1} + \chi_z \hat{E}_t^{CB} z_{t+1}. \tag{4}$$

Again the same notation without the "ˆ" and superscript CB will denote RE (of the central bank). The rule (4) with private expectations in place of central bank forecasts has been considered in the literature; either as a version of an *ad hoc* Taylor (or instrument) rule, see e.g. (Bullard and Mitra 2002)

[1] (Sargent 1999) and (Cho, Williams, and Sargent 2002) study a model of the natural rate hypothesis and a misspecification by the central bank.

or as an expectations based optimal discretionary policy as in (Evans and Honkapohja 2003b). We assume $\chi_z \geq 0$ and $\chi_\pi \geq 0$ throughout the paper.

The reduced form of (1), (2), (3), and (4) is

$$\begin{aligned} y_t &= D + A^P \hat{E}_t^P y_{t+1} + A^{CB} \hat{E}_t^{CB} y_{t+1} + B w_t, \\ w_t &= F w_{t-1} + v_t. \end{aligned} \tag{5}$$

where $y_t = (z_t, \pi_t)'$, $w_t = (g_t, u_t)'$ and

$$\begin{aligned} D &= \begin{pmatrix} -\varphi \\ -\lambda\varphi \end{pmatrix} \chi_0,\ A^P = \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix},\ A^{CB} = \begin{pmatrix} -\varphi\chi_z & -\varphi\chi_\pi \\ -\lambda\varphi\chi_z & -\lambda\varphi\chi_\pi \end{pmatrix} \\ & B = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}. \end{aligned}$$

3 Asymmetric Information between the Private Sector and Central Bank

Using a standard approach to learning discussed in the treatise (Evans and Honkapohja 2001), (Honkapohja and Mitra 2003) study this model of learning for various cases of heterogenous learning. They establish constraints for the policy parameters that need to be fulfilled in order to have stability of the resulting REE under adaptive learning. Here we consider heterogeneity in forecasting arising from differences in the information sets of the private agents and the central bank.

We take up only one case in which one party observes only one of shocks while the other sees both of them. We develop the formal analysis when the central bank does not see u_t and, moreover, does not have a good signal about it. If the private sector observes both shocks, its perceived law of motion (PLM) and forecasts have the form

$$\begin{aligned} y_t &= a^P + b^P w_t, \text{ where } a^P = \begin{pmatrix} a_z^P \\ a_\pi^P \end{pmatrix} \text{ and } b^P = \begin{pmatrix} b_{zg}^P & b_{zu}^P \\ b_{\pi g}^P & b_{\pi u}^P \end{pmatrix} \\ \hat{E}_t^P y_{t+1} &= a^P + b^P F w_t. \end{aligned}$$

The central bank guesses that the values of the endogenous variables depend just on the aggregate demand shock g_t. The PLM and the forecast function of the central bank have the form

$$
\begin{aligned}
y_t &= a^{CB} + b^{CB} g_t, \text{ where } a^{CB} = \begin{pmatrix} a_z^{CB} \\ a_\pi^{CB} \end{pmatrix} \text{ and } b^{CB} = \begin{pmatrix} b_z^{CB} \\ b_\pi^{CB} \end{pmatrix}, \\
\hat{E}_t^{CB} y_{t+1} &= a^{CB} + b^{CB} \mu g_t.
\end{aligned}
$$

The forecast function of the central bank does not nest the symmetric information REE, i.e. the PLM of the central bank is misspecified even asymptotically. However, the economy may converge to some equilibrium that is rational in a limited information sense. These restricted perceptions equilibria (RPE) are studied in Chapter 13 of (Evans and Honkapohja 2001).

Substituting the resulting forecast functions into (5), the actual law of motion (ALM) is

$$
\begin{aligned}
y_t = {} & \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix} a^P + \begin{pmatrix} -\varphi\chi_z & -\varphi\chi_\pi \\ -\lambda\varphi\chi_z & -\lambda\varphi\chi_\pi \end{pmatrix} a^{CB} + \\
& \left[\begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix} b_g^P + \begin{pmatrix} -\varphi\chi_z & -\varphi\chi_\pi \\ -\lambda\varphi\chi_z & -\lambda\varphi\chi_\pi \end{pmatrix} b^{CB} \right] \mu g_t + \\
& \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix} b_u^P \rho u_t + \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} g_t \\ u_t \end{pmatrix},
\end{aligned}
$$

where b_g^P and b_u^P are, respectively, the 1st and 2nd columns of matrix b^P. We can write this formally as

$$
y_t = A^P a^P + A^{CB} a^{CB} + \left[\mu \left(A^P b_g^P + A^{CB} b^{CB} \right) + B_g \right] g_t + (\rho A^P b_u^P + B_u) u_t, \tag{6}
$$

where B_g, B_u are the columns of B defined in (5).

The parameters for both PLMs are assumed to be updated by recursive least squares. The RLS algorithm for the private sector takes the form

$$
\begin{aligned}
(\phi_t^P)' &= (\phi_{t-1}^P)' + t^{-1} (R_t^P)^{-1} x_{t-1} (y_{t-1} - \phi_{t-1}^P x_{t-1})' \qquad (7) \\
R_t^P &= R_{t-1}^P + t^{-1} [x_{t-1} (x_{t-1})' - R_{t-1}^P].
\end{aligned}
$$

Introducing the notation $\xi_t^{CB} = (a_t^{CB}, b_t^{CB})$ and $(x_t^{CB})' = (1, g_t)$, the estimation algorithm for the central bank takes the form

$$
\begin{aligned}
(\xi_t^{CB})' &= (\xi_{t-1}^{CB})' + t^{-1} (R_t^{CB})^{-1} x_{t-1}^{CB} (y_{t-1} - \xi_{t-1}^{CB} x_{t-1}^{CB})', \qquad (8) \\
R_t^{CB} &= R_{t-1}^{CB} + t^{-1} [x_{t-1}^{CB} (x_{t-1}^{CB})' - R_{t-1}^{CB}].
\end{aligned}
$$

This formulation is similar to that of the algorithm of the private sector, except that u_t does not appear in the state variables x_t^{CB}.

The RPE is given by the solution to the equations

$$\begin{aligned}
a^P &= A^P a^P + A^{CB} a^{CB}, \\
a^{CB} &= A^P a^P + A^{CB} a^{CB}, \\
b^P &= \left[\mu(A^P b_g^P + A^{CB} b^{CB}) + B_g, \rho A^P b_u^P + B_u\right], \\
b^{CB} &= \mu(A^P b_g^P + A^{CB} b^{CB}) + B_g.
\end{aligned}$$

We have the stability result:

Proposition 1 *The RPE is locally stable under learning if all eigenvalues of the following two matrices*

$$\begin{pmatrix} A^P - I & A^{CB} \\ A^P & A^{CB} - I \end{pmatrix}, \rho A^P - I.$$

have negative real parts.

The proof is in the Appendix. The first condition in Proposition 1 is simply the E-stability requirement for the full information REE. The second condition requires the autocorrelation coefficient ρ in the cost push shocks to be sufficiently small since A^P has an eigenvalue bigger than one (the other one being between 0 and 1). Computing the eigenvalues of A^P, we get:

Corollary 2 *The RPE is locally stable under learning iff*

$$\begin{aligned}
(1-\beta)\chi_z + \lambda(\chi_\pi - 1) &> 0, \\
(2\beta)^{-1}[1 + \beta + \lambda\varphi - \sqrt{(1+\beta+\lambda\varphi)^2 - 4\beta}] &\geq \rho.
\end{aligned}$$

The first condition is just the Taylor principle that characterizes stability under learning if both the private sector and the central bank have identical learning rules, see (Bullard and Mitra 2002). The second condition can be restrictive, especially when there is high persistence in the u_t shock.[2]

The results are quite different if, in contrast, the private sector observes less than the central bank.

[2] If instead the central bank observes (only) the u_t shock but not the g_t shock, then the condition for stability is the same as above with μ replacing ρ in Corollary 2. If the CB observes neither shock, then the stability conditions also include the same upper bound for μ.

Proposition 3 *The stability condition reduces to the standard requirement*

$$(1-\beta)\chi_z + \lambda(\chi_\pi - 1) > 0$$

in either case of non-observability of g_t or u_t by the private sector.

The proof follows by interchanging the roles of A^P and A^{CB} in Proposition 1 and noting that the eigenvalues of A^{CB} are non-positive.

4 Concluding Comments

The preceding results suggest that lack of information on the part of the central bank can lead to problems of instability under learning more easily than when information is symmetric. Corollary 2 indicates that, in the case of central bank having less information about the shocks than the private sector, there is a further learnability constraint that must be met by the policy maker in addition to the learnability constraint that arises in the symmetric information setting. In contrast, if the private sector has less information than the central bank, then Proposition 3 indicates that no further learnability constraint arises as a result of the asymmetric information.

Our results support the general notion that the central bank should spend enough resources in acquiring good information about the shocks hitting the economy. There is recent empirical evidence that the Federal Reserve appears to possess information about the current and future state of the economy that is not known to commercial forecasters, see (Romer and Romer 2001).

5 Appendix: Proof of Proposition 1

Using the methodology in Section 13.1.1 of (Evans and Honkapohja 2001) we compute the associated differential equation (ODE) for the algorithms (7) and (8)

$$\begin{aligned}
& E(R^P)^{-1}x_{t-1}(y_{t-1}-\phi^P x_{t-1})' \\
= \; & E(R^P)^{-1}\begin{pmatrix} 1 \\ g_{t-1} \\ u_{t-1} \end{pmatrix}\left[(z_{t-1},\pi_{t-1}) - (1, g_{t-1}, u_{t-1})(\phi^P)'\right] \\
= \; & E(R^P)^{-1}\begin{pmatrix} 1 \\ g_{t-1} \\ u_{t-1} \end{pmatrix}\left[(1, g_{t-1}, u_{t-1})\left[\begin{pmatrix} \tilde{a}_z & \tilde{a}_\pi \\ \tilde{A}_{zg} & \tilde{A}_{\pi g} \\ \tilde{A}_{zu} & \tilde{A}_{\pi u} \end{pmatrix} - (\phi^P)'\right]\right],
\end{aligned}$$

where the temporary notation $\tilde{a}_z$, $\tilde{A}_{zg}$ etc. is obtained from

$$(z_{t-1}, \pi_{t-1}) = (1, g_{t-1}, u_{t-1}) \begin{pmatrix} \tilde{a}_z & \tilde{a}_\pi \\ \tilde{A}_{zg} & \tilde{A}_{\pi g} \\ \tilde{A}_{zu} & \tilde{A}_{\pi u} \end{pmatrix}.$$

Taking expectations and limits the ODE for the private sector becomes

$$\begin{aligned} d\phi^P/d\tau &= (R^P)^{-1}(Exx')[T^P(\phi^P, \phi^{CB}) - \phi^P] \\ dR^P/d\tau &= Exx' - R^P, \end{aligned} \tag{9}$$

where $Exx' = \lim_t Ex_t x_t{}'$ and

$$T^P(\phi^P, \phi^{CB}) = (A^P a^P + A^{CB} a^{CB}, \mu(A^P b_g^P + A^{CB} b^{CB}) + B_g, \rho A^P b_u^P + B_u).$$

The ODE for the algorithm of the central bank is analogously

$$\begin{aligned} & E(R^{CB})^{-1} x_{t-1}^{CB} (y_{t-1} - \phi^{CB} x_{t-1}^{CB})' \\ = \; & E(R^{CB})^{-1} \begin{pmatrix} 1 \\ g_{t-1} \end{pmatrix} (1, g_{t-1}) \left[\begin{pmatrix} \tilde{a}_z & \tilde{a}_\pi \\ \tilde{A}_{zg} & \tilde{A}_{\pi g} \end{pmatrix} - (\phi^{CB})' \right] \\ & + E(R^{CB})^{-1} \begin{pmatrix} 1 \\ g_{t-1} \end{pmatrix} u_{t-1} (\tilde{A}_{zu}, \tilde{A}_{\pi u}). \end{aligned}$$

Since g_t and u_t are uncorrelated and have zero means, the second term in this expression is zero. The ODE for the central bank is then

$$\begin{aligned} d\phi^{CB}/d\tau &= (R^{CB})^{-1}(Ex^{CB}(x^{CB})')[T^{CB}(\phi^P, \phi^{CB}) - \phi^{CB}] \\ dR^{CB}/d\tau &= Ex^{CB}(x^{CB})' - R^{CB}, \end{aligned} \tag{10}$$

where $Ex^{CB}(x^{CB})' = \lim_t Ex_t^{CB}(x_t^{CB})'$ and

$$T^{CB}(\phi^P, \phi^{CB}) = (A^P a^P + A^{CB} a^{CB}, \mu(A^P b_g^P + A^{CB} b^{CB}) + B_g).$$

Local stability of the associated differential equations (9) and (10) is governed by the local stability of the "small" differential equations

$$\begin{aligned} d\phi^P/d\tau &= T^P(\phi^P, \phi^{CB}) - \phi^P \\ d\phi^{CB}/d\tau &= T^{CB}(\phi^P, \phi^{CB}) - \phi^{CB}, \end{aligned}$$

which are the modified E-stability differential equations. Inspecting the T^P and T^{CB} mappings it is seen that for constant terms a^P and a^{CB}, as well as for the terms b_g^P and b^{CB}, standard E-stability arguments apply, see (Honkapohja and Mitra 2003) while the E-stability equation for b_u^P is simply

$$db_u^P/d\tau = (\rho A^P - I)b_u^P + B_u,$$

which completes the proof.

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