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Mixed up? That's good for motivation*

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Abstract

An essential ingredient in models of career concerns is ex ante uncertainty about an agent's type. This paper shows how career concerns can arise even in the absence of any such ex ante uncertainty, if the unobservable actions that an agent takes influence his future productivity. By implementing effort in mixed strategies the principal can endogenously induce uncertainty about the agent's ex post productivity and generate reputational incentives. Our main result is that creating such ambiguity can be optimal for the principal, even though this exposes the agent to additional risk and reduces output. This finding demonstrates the importance of mixed strategies in contracting environments with imperfect commitment, which contrasts with standard agency models where implementing mixed strategy actions typically is not optimal if pure strategies are also implementable.

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1 Introduction

To a large extent incentives in organizations arise both from workers' career concerns and from explicit incentive schemes. A worker faced with a labor market that tries to infer his ability from observations of his past performance has an incentive to provide some effort to influence this updating process (even in the absence of any explicit monetary rewards). An essential ingredient in extant models of such career concerns is the existence of some ex ante uncertainty about a worker's type[1]. In this paper we demonstrate how reputational incentives can arise endogeneously, when there is no such ex ante uncertainty regarding abilities, but the worker's actions affect his future productivity through learning by doing. The key insight is that in such a setting reputation is a function of the market's perception of what actions the worker has taken. As a consequence, the firm can strategically affect the market's updating process through the design of its explicit incentive contract. Our main result is that it can be optimal for the firm to create ambiguity[2] about the actions that a worker has taken, even though this exposes risk averse workers to additional risk and reduces the firm's expected output.

We derive our results in a simple two-period model of human capital acquisition. In the first period, a principal ("she") contracts with an agent ("he"), whose productivity is common knowledge, to induce unobservable effort through a spot contract. The agent's effort not only stochastically increases observable output but, through learning by doing, also deterministically raises the agent's unobservable human capital at the end of the first period. In the second period, the principal competes with other potential employers, who all update their estimates about the agent's human capital based on the contract signed and the output produced in this prior contractual relation.

The main argument in the paper can be summarized as follows. If a contract implements effort in pure strategies then learning by doing always leads to high human capital and, in equilibrium, the agent's reputation is independent of output. In contrast, the principal can create reputational incentives by inducing mixed strategy effort since, under such a scheme, high output leads to a larger second-period wage than low output. High output is evidence that the agent actually exerted effort and acquired high human capital. In contrast, for low output, it is likely that the agent did not exert effort and has low human capital. Implementing mixed strategy effort is optimal for the principal if the gain from reduced pay due to these

[1] For example, see Borland (1992).

[2] Our approach to "ambiguity" is different from the one in Bernheim and Whinston (1998). They show that a principal faced with contractual incompleteness may voluntarily leave contracting parties' obligations vague or unspecified, resulting in contracts that are more incomplete than necessary.

reputational incentives outweighs the expected loss in output due to the lower probability of effort provision.

Several extensions of the base model are considered. We apply the model to analyze how much focus on specific tasks contracts should mandate. Because reputational incentives are affected by an agent's focus it may be optimal for the principal to be vague on the type of task that the agent should pursue. Moreover, we address the issue of optimal screening of job applicants. In a setting where a perfect screening technology is costless we derive conditions under which a principal optimally refrains from fully screening heterogeneous job applicants.

In agency models it is typically not optimal for the principal to induce agents to play a mixed strategy if a pure strategy can be implemented.[3] However, several papers on contracting under *asymmetric information* demonstrate that mixed strategies can be optimal when a contractual incompleteness prevents the dynamic contracting problem from collapsing to one that is essentially static. Laffont and Tirole (1988) produce such a result in a setting where a principal with limited commitment power repeatedly contracts with the same agent to create incentives in a moral hazard problem. Contracting is complicated by a ratchet effect[4], and this induces the principal to implement mixed strategies for agents rather than fully revealing pure strategies. Bester and Strausz (2001) extend the revelation mechanism to account for imperfect commitment powers of the mechanism designer and show that, under an optimal mechanism, the agent does not reveal his type with certainty. Our results demonstrate that, even when principal and agent contract under *symmetric information*, implementation of a mixed rather than a pure strategy can be optimal.

Our paper is also related to Dewatripont, Jewitt, and Tirole (1999a) who show that, in the context of Holmström (1982/99)'s model of pure career concerns, reputational incentives can increase as the signal structure becomes coarser. Similar results obtain when the design of explicit incentives interact with career concerns (Koch and Peyrache (2003a, 2003b)) or with ratchet effects (Meyer and Vickers 1997). A new contribution of our paper is that it shows how a principal can design explicit incentive schemes to create reputational incentives through ambiguity about an agent's future productive value, even when agents are ex ante homogeneous.

The paper is organized as follows. Section 2 introduces the model. The limited liability case is then analyzed in section 3. Section 4 treats the case with unlimited liability. In section 5

[3]There exist several papers in which the principal can only implement mixed strategies (e.g., Fudenberg and Tirole (1990) or Khalil (1997)).

[4]If the agent reveals that he is a low cost type he will face a tougher incentive scheme in the next period than if he reveals to be a high cost type.

we discuss how the base model can be reinterpreted and applied to a multi-task setting as well as to determine the optimal amount of screening of job applicants. Section 6 concludes.

2 Model

We consider a two-period model where, in the first period, an employee (the agent), whose ability is common knowledge, contracts with a risk-neutral firm (the principal) over the production of output $\tilde{y} \in \{\underline{y}, \bar{y}\}$. In the absence of effort (e=0), the agent produces $\underline{y}$ with probability one and accumulates human capital $\underline{H}$. By exerting unobservable effort e=1 at private cost ψ, he produces $\bar{y} \equiv \underline{y} + \Delta\, y$ with probability $\pi \in (0,1)$ and $\underline{y}$ with probability $1-\pi$. In addition, he then acquires human capital $\bar{H} \equiv \underline{H} + \Delta\, H$. A fraction α of this human capital is firm specific, whereas the rest is general. If he does not accept the job the agent faces an outside option that offers a life-time utility of zero.

Neither the agent's effort nor his human capital are observed. However, the contract signed with the agent is publicly observable, and at the end of the first period, all the parties (the agent, the principal and other potential employers) get to know the agent's level of output[5]. As a consequence, in the second period, the first principal competes à la Bertrand with other principals for the agent under symmetric information. If the agent stays with the first principal, he produces output equal to the level of total human capital H accumulated in the first period. In contrast, when switching to a new principal, the agent can only produce $(1-\alpha)\, H$. Thus, it is straightforward that in the equilibrium of the second-period continuation game, the agent stays with the first principal, who matches outside offers. That is, the agent earns a second-period wage t_2 equal to his expected general human capital conditional on the contract and on realized output in the first period:[6]

$$t_2(\tilde{y}) = (1-\alpha)\, E\left[H|\tilde{y}, contract\right]. \tag{1}$$

As is standard in models of career concerns, we assume that the first principal can only offer the agent a contract that makes first-period transfers t_1 contingent on output in that period (spot contract).[7]

[5] For an analysis where the principal can decide on what type of performance information to reveal to the market and the repercussions that this has on the design of incentive schemes see Koch and Peyrache (2003a).

[6] Our results go through under any formulation in which the second-period rent accruing to the agent is increasing in his expected human capital.

[7] Typically, in labor markets parties lack full pre-commitment power (e.g., workers cannot cede their right to revoke a contract because slavery is forbidden). If the parties had such powers the dynamic incentive problem would essentially collapse to a static one in which reputational incentives do not matter.

The agent's preferences are represented by the following time-separable utility function over first- and second-period transfers:

$$U(e,\tilde{y}) = u(t_1(\tilde{y})) + u(t_2(\tilde{y})) - e \cdot \psi, \tag{2}$$

where $u : \mathbb{R} \to \mathbb{R}$, $u(0) = 0$ and $u' > 0$. The agent can either be risk neutral or risk averse, i.e., we assume that $u'' \leq 0$. Finally, let g be the inverse function of u.

3 Limited Liability

In this section we analyze the model under the assumption that the agent is protected by limited liability, i.e., $t_\tau \geq 0$, $\tau = 1, 2$.

3.1 Implementing effort in pure strategies

To construct a pure strategy Perfect Bayesian Equilibrium where the agent exerts effort, fix the beliefs of the agent and the market as follows: market beliefs are that the agent exerts effort in the first period under the observed contract, and the agent correctly anticipates this when taking his effort decision. This yields him second-period utility[8]

$$u_2^P \equiv u(t_2^P(\bar{y})) = u(t_2^P(\underline{y})) = u\left((1-\alpha)\,\bar{H}\right). \tag{3}$$

If the principal wants to implement effort $e = 1$, she has to respect the following incentive and individual rationality constraints:

$$\pi\,\bar{u}_1^P + (1-\pi)\,\underline{u}_1^P - \psi + u_2^P \;\; \geq \;\; \underline{u}_1^P + u_2^P, \tag{IC:P}$$

$$\pi\,\bar{u}_1^P + (1-\pi)\,\underline{u}_1^P - \psi + u_2^P \;\; \geq \;\; 0, \tag{IR:P}$$

where $\bar{u}_1^P \equiv u(t_1^P(\bar{y}))$ and $\underline{u}_1^P \equiv u(t_1^P(\underline{y}))$. Since $t_1^P(\tilde{y}) \geq 0$, $u(0) = 0$, $u' > 0$, and $u_2^P > 0$, constraint (IR:P) cannot bind when constraint (IC:P) is satisfied, thus $\underline{u}_1^P = 0$ and $\bar{u}_1^P = \frac{\psi}{\pi}$. This yields transfers $\underline{t}^P = g\left(\underline{u}_1^P\right) = 0$ and $\bar{t}^P = g\left(\bar{u}_1^P\right) = g\left(\frac{\psi}{\pi}\right)$.
Under such a contract the principal's expected profit is:

$$\Pi^P = \underline{y} + \pi\left[\Delta\,y - g\left(\frac{\psi}{\pi}\right)\right] + \alpha\bar{H}. \tag{4}$$

Since pure strategy implementation will be our benchmark case we assume that the gain in output for the principal, $\Delta\,y$, exceeds the monetary transfer to the agent that is necessary to induce pure strategy effort, $g\left(\frac{\psi}{\pi}\right)$:

[8] The superscript P refers to pure strategy implementation, and is used later in comparisons with mixed strategy implementation, carrying superscript M.

Assumption 1 $\Delta y > g\left(\frac{\psi}{\pi}\right)$.

In equilibrium, the market anticipates that the agent will act according to the contract that is offered to him. That is, it assumes that the agent exerts effort and therefore always acquires human capital $\bar{H}$. This implies that market beliefs do not depend on the first-period performance of the agent. Given that the employee will be considered to have high human capital regardless of his output, all the incentives in the first period have to be provided through monetary transfers.

3.2 Implementing effort in mixed strategies

Suppose now that the market's beliefs are that the agent exerts effort with probability p and no effort otherwise under the first-period contract. Then, the market's expectation of the agent's level of human capital becomes a function of the first-period output $\tilde{y}$. Correctly anticipating these beliefs, the agent faces one of the following second-period utilities, depending on the realized output:[9]

$$\bar{u}_2^M \equiv u(t_2^M(\bar{y})) = u\left((1-\alpha)\,\bar{H}\right), \tag{5}$$

$$\underline{u}_2^M(p) \equiv u(t_2^M(\underline{y})) = u\left((1-\alpha)\,\left[\underline{H} + \frac{p\,(1-\pi)}{1-p\,\pi}\,\Delta H\right]\right). \tag{6}$$

Given these beliefs, to implement mixed strategy p the principal's contract must be incentive compatible, i.e., it must satisfy

$$\pi\,\left(\bar{u}_1^M(p) + \bar{u}_2^M\right) + (1-\pi)\left(\underline{u}_1^M + \underline{u}_2^M(p)\right) - \psi = \underline{u}_1^M + \underline{u}_2^M(p). \tag{IC:M}$$

This incentive constraint implies that the individual rationality constraint,

$$\underline{u}_1^M + \underline{u}_2^M(p) \geq 0, \tag{IR:M}$$

is also satisfied. As before, the principal sets $\underline{u}_1^M = 0$ and thus $\underline{t}^M = g(0) = 0$. From (IC:M) and the limited liability constraint we then obtain

$$\bar{u}_1^M(p) = \max\left\{\frac{\psi}{\pi} - \left(\bar{u}_2^M - \underline{u}_2^M(p)\right),\, 0\right\}. \tag{7}$$

Thus, monetary transfers are lower than with pure strategy implementation: for all $p \in (0,1)$ we have $0 \leq \bar{t}^M(p) = g\left(\bar{u}_1^M(p)\right) < \bar{t}^P$. For our argument it is sufficient to focus on the case where the limited liability constraint is not violated for any $p \in (0,1)$ and $\alpha \in (0,1)$. Since $\bar{u}_2^M \leq u\left(\bar{H}\right)$ and $\underline{u}_2^M(p) > u\left(\underline{H}\right)$ for any $p \in (0,1)$ the following condition is sufficient to ensure that $\bar{t}^M(p) > 0$ for all possible values (α, p):

[9] In the following, to simplify the exposition, the dependence on p is dropped whenever functions are constant in p.

Assumption 2 $\frac{\psi}{\pi} \geq u\left(\bar{H}\right) - u\left(\underline{H}\right).$

The principal's expected profit for $p \in (0,1)$ is given by:

$$\Pi^M(p) = \underline{y} + p\,\pi\,\left[\Delta\,y - g\left(\bar{u}_1^M(p)\right)\right] + \alpha\left[p\,\bar{H} + (1-p)\,\underline{H}\right]. \tag{8}$$

Our model incorporates "on-the-job human capital acquisition". Therefore, whenever the market cannot perfectly infer the agent's effort level from the contract offered to the agent, it has to rely on the realized output as a signal for the effort actually exerted. A high level of output indicates that effort was exerted and that high human capital was acquired by the agent (cf. equation (5)). In contrast, a low level of output is bad news about human capital since the market is no longer certain that the agent exerted effort (cf. equation (6)). This creates a reputational wedge $\bar{u}_2^M - \underline{u}_2^M(p)$ that the firm can exploit to lower monetary transfers. Therefore, incentives for effort arise even under rather low-powered monetary incentive schemes.

3.3 Optimal contract

The principal's gain from implementing mixed strategy $p \in (0,1)$ instead of pure strategy $p = 1$ is

$$G(p) \equiv \Pi^M(p) - \Pi^P = \underbrace{\pi\left[g\left(\frac{\psi}{\pi}\right) - p\,\bar{t}^M\right]}_{\text{saved implementation cost}} - \underbrace{(1-p)\,\left[\pi\,\Delta\,y + \alpha\Delta\,H\right].}_{\text{loss in expected output}} \tag{9}$$

The principal implements effort in mixed strategies whenever there exists a probability $p \in (0,1)$ such that the saved expected implementation cost outweighs the expected loss in output relative to pure strategy effort. The following result gives a sufficient condition under which a contract that implements a mixed strategy is optimal:

Proposition 1

Under limited liability of the agent, a sufficient condition for the profit maximizing equilibrium contract to involve a mixed strategy $p^* \in (0,1)$ is

$$\underbrace{\pi\left(\Delta\,y - g\left(\frac{\psi}{\pi}\right)\right) + \alpha\Delta H}_{\textit{marginal loss in output for } p \uparrow 1} < \underbrace{\pi(1-\alpha)\,Z\frac{\Delta H}{1-\pi}}_{\textit{marginal gain in cost reduction for } p \uparrow 1} \tag{10}$$

where $Z \equiv g'\left(\frac{\psi}{\pi}\right)\,u'\left((1-\alpha)\,\bar{H}\right)$.

Proof.

Taking the derivative of $G(p)$ with respect to p, we get

$$\begin{aligned}\frac{d\,G(p)}{d\,p} &= \pi\,\Delta\,y + \alpha\,\Delta\,H \\ &- \quad \pi\,g\left(\bar{u}_1^M(p)\right) - p\,\pi\,g'\left(\bar{u}_1^M(p)\right)\,u'\left((1-\alpha)\,\left[\underline{H} + \frac{p\,(1-\pi)}{1-p\,\pi}\,\Delta\,H\right]\right)\,(1-\alpha)\frac{1-\pi}{(1-p\,\pi)^2}\,\Delta\,H.\end{aligned} \tag{11}$$

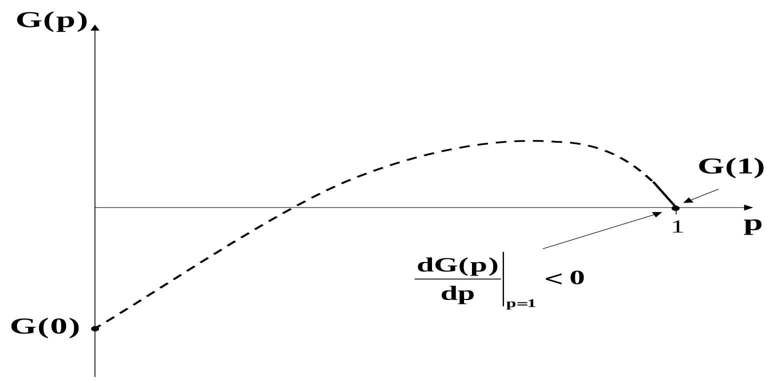

Figure 1: Illustration of proof.

Given that $G(0) = \pi \left(g\left(\frac{\psi}{\pi}\right) - \Delta\, y\right) - \alpha\, \Delta\, H < 0$ (by assumption 1) and $G(1) = 0$, a sufficient condition for an interior solution is that $\left.\frac{d\,G(p)}{d\,p}\right|_{p=1} < 0$ (see figure 1). ■

To understand the trade-off between inducing effort provision and creating reputational incentives it is useful to first consider the comparative statics of the result. The larger the difference in first-period outputs $\Delta\, y$ and the firm-specific component of human capital α, the less likely that the principal implements mixed strategy effort. When $\alpha = 1$, human capital is fully firm specific and the agent earns nothing in the second period. Consequently, there are no reputational incentives because the agent's second-period utility does not depend on first-period actions. Therefore, the optimal contract implements pure strategy effort. The other polar case is when all the human capital is general, i.e., $\alpha = 0$. Then the impact of first-period actions on the agent's second-period utility is maximal since he earns the entire return to the human capital acquired in the first period. The condition for the optimality of a mixed strategy contract becomes: $\Delta\, y - g\left(\frac{\psi}{\pi}\right) < g'\left(\frac{\psi}{\pi}\right)\, u'\left(\bar{H}\right)\, \frac{\Delta H}{(1-\pi)}$. Given that $u(\cdot)$ and $g(\cdot)$ are increasing, clearly the set of values of Δy that satisfy both assumption 1 and condition (10) is non-empty. In contrast, the effect of an increase in the human capital return to effort, ΔH, depends on the sign of the expression in square brackets on the right-hand side (RHS) of inequality (10). An increase in ΔH raises both the expected loss in firm-specific human capital that the firm incurs by not implementing pure strategy effort, and the amount of cost savings due to reputational incentives. Such an increase makes mixed strategy implementation more likely the lower is the value of α. Indeed, for $\alpha = 0$, an increase of ΔH has no effect on the marginal loss (LHS) but has a large effect on the marginal gain in cost reduction (RHS). In contrast, for $\alpha = 1$ the cost saving effect is dominated and mixed strategy implementation becomes less likely.

An interesting feature of Proposition 1 is that the principal faces a trade-off between the riskiness of output and effort incentives, which to our knowledge is a new result in a model with explicit incentives and risk averse agents.

In our framework, the agent earns a limited liability rent because the principal cannot make him pay for the increase in future earnings due to accumulated human capital. This rent is $u\left((1-\alpha)\,\bar{H}\right)$ in the case of pure strategy implementation and $u\left((1-\alpha)\,\left[\underline{H}+\frac{p\,(1-\pi)}{1-p\,\pi}\,\Delta\,H\right]\right)$ in the case of mixed strategy implementation. Following our analysis above, optimal contracts only involve mixed strategy effort if the limited liability rent is strictly positive, i.e., human capital is not fully specific ($\alpha < 1$). In such a setting, due to competition in the second-period labor market, the agent captures part of the productivity gains from learning by doing since limited liability prevents the principal from fully extracting rents in the first period. Remarkably, proposition 1 shows that, for appropriate values of the parameters, the principal exposes a risk averse agent to more risk than in a static explicit incentive model: the lottery over transfers faced by an agent under a contract that implements pure strategy effort (which corresponds to the static moral hazard contract) first-order stochastically dominates the one faced under a contract with mixed strategy effort. Such exposure to risk creates uncertainty about the agent's second-period productivity and generates reputational incentives that reduce the limited liability rent. The uncertainty about the acquired human capital induced by mixed strategy effort provision is a means to transform a homogeneous group of unexperienced agents into a heterogeneous group of experienced agents for whom reputational incentives exist.[10]

While mixed strategy implementation may enhance the principal's profits it always reduces welfare compared to pure strategy implementation. Overall, mixed strategy effort decreases welfare by lowering the probability of high first-period output[11] and reducing second-period output because less human capital is accumulated. The principal can profitably create such a distortion since, when implementing mixed strategies, she does not internalize the second-period utility loss $u\left((1-\alpha)\,\bar{H}\right)-u\left((1-\alpha)\,\left[\underline{H}+\frac{p\,(1-\pi)}{1-p\,\pi}\,\Delta\,H\right]\right)$ that the agent is subject to following a low state realization in the first period.

4 Unlimited liability

In the absence of limited liability, the principal can implement pure strategy effort and extract the agent's gain in future earnings due to accumulated human capital through transfers, by

[10]Creating ambiguity about agents' types might be optimal for other reasons as well if there are heterogeneous unexperienced agents (e.g., Koch and Peyrache (2003a,2003b)).

[11]Recall that it is efficient for the agent to exert effort (by assumption 1).

setting $\bar{u}_1^P = \frac{\psi}{\pi} - u\left((1-\alpha)\,\bar{H}\right)$ and $\underline{u}_1^P = -u\left((1-\alpha)\,\bar{H}\right)$. This yields expected profit:

$$\hat{\Pi}^P = \underline{y} + \pi\,\left[\Delta\,y - g\left(\bar{u}_1^P\right)\right] - (1-\pi)\,g\left(-u\left((1-\alpha)\,\bar{H}\right)\right) + \alpha\,\bar{H}. \tag{12}$$

In contrast, under a contract that implements a mixed strategy p the principal sets utility levels $\bar{u}_1^M = \bar{u}_1^P = \frac{\psi}{\pi} - u\left((1-\alpha)\,\bar{H}\right)$ and $\underline{u}_1^M(p) = -\underline{u}_2^M(p) = -u\left((1-\alpha)\,\left[\underline{H} + \frac{p\,(1-\pi)}{1-p\,\pi}\,\Delta\,H\right]\right)$, yielding expected profit

$$\hat{\Pi}^M(p) = \underline{y} + p\,\pi\,\left[\Delta\,y - g\left(\bar{u}_1^P\right)\right] - (1-p\,\pi)\,g\left(\underline{u}_1^M(p)\right) + \alpha\,\left[p\,\bar{H} + (1-p)\,\underline{H}\right]. \tag{13}$$

Implementing a mixed strategy instead of a pure strategy has two effects. First, since $0 > g\left(\underline{u}_1^M(p)\right) > g\left(\underline{u}_1^P\right)$, the payment received by the principal in the low-output state decreases (note that the payoff in the high-output state does not change). Second, the probability of a high- (low-)output state decreases (increases). The overall effect unambiguously decreases the principal's profit if the payoff from pure strategy implementation received by a principal in the high-output state exceeds that received in the low-output state, i.e., if

$$\Delta\,y - \underbrace{g\left(\frac{\psi}{\pi} - u\left((1-\alpha)\,\bar{H}\right)\right)}_{\equiv g\left(\bar{u}_1^P\right)} > -\underbrace{g\left(-u\left((1-\alpha)\,\bar{H}\right)\right)}_{\equiv g\left(\underline{u}_1^P\right)}. \tag{14}$$

Under this condition the equilibrium contract always implements effort in pure strategies. This allows us to prove the following result:

Proposition 2

Under unlimited liability of the agent, the principal always induces effort in pure strategies.

Proof.

The proof is by contradiction. Suppose that condition (14) is violated (which is a necessary condition for mixed strategy implementation to be optimal):

$$\begin{aligned} -\,g\left(-u\left((1-\alpha)\,\bar{H}\right)\right) \;&\geq\; \Delta\,y - g\left(\frac{\psi}{\pi} - u\left((1-\alpha)\,\bar{H}\right)\right) & (15)\\ &>\; g\left(\frac{\psi}{\pi}\right) - g\left(\frac{\psi}{\pi} - u\left((1-\alpha)\,\bar{H}\right)\right), & (16)\end{aligned}$$

where the last relation follows from assumption 1. Rewriting the last expression we get

$$g\left(\frac{\psi}{\pi}\right) - g\left(\frac{\psi}{\pi} - u\left((1-\alpha)\,\bar{H}\right)\right) = \int_{\frac{\psi}{\pi}-u\left((1-\alpha)\,\bar{H}\right)}^{\frac{\psi}{\pi}} g'(q)\,dq \tag{17}$$

$$\geq \int_{-u\left((1-\alpha)\,\bar{H}\right)}^{0} g'(q)dq = -g\left(-u\left((1-\alpha)\,\bar{H}\right)\right),$$

since $g''(x) \geq 0\ \forall x \in \mathbb{R}$. This leads to a contradiction. ■

The intuition for the result is clear when the agent is risk neutral. In the absence of limited liability, the principal can extract the entire rents accruing to the agent in the second period, e.g., by selling the project to the agent and making him the residual claimant. Pure strategy effort is optimal since it maximizes the surplus accruing from first-period production and human capital acquisition.

In the case of a risk averse agent, the principal faces a trade-off between incentives and insurance when offering the agent an incentive contract. However, the nature of this trade-off is different from the standard one in static models since the contract affects incentives indirectly through market beliefs about the level of effort that the agent exerted and, thus, determines the agent's wage in the second period. Our result confirms the intuition that the principal does not expose a risk averse agent to the additional risk associated with mixed strategy effort when she can make him pay for acquired human capital.[12]

5 Alternative Interpretations and Extensions

In the previous sections we derived conditions under which a principal benefits from implementing mixed strategy effort and thereby creating ambiguity about the actions that an agent has taken. To illustrate how the base model can be used as a building block in applications we present two simple extensions that analyze the issue of optimal mission focus and optimal screening of job seekers. The latter extension demonstrates that our main insights do not necessarily require mixing over actions by the agent.

5.1 Multiple Tasks

In this section we allow for N different tasks, which yield output $\tilde{y}_i \in \{\underline{y}, \bar{y}\}$, $i = 1, ..., N$ according to the same production technology as in the base model. Through learning by doing in task i the agent accumulates task-specific human capital H_i that is fully transferable to other firms interested in this type of human capital. The marketability of task-specific human capital depends on market forces that are beyond control of the firm and is uncertain. To capture this, we assume that the value of human capital in task i in the second-period labor market is $H_i = \bar{H}\, x_i$, where x_i is independently distributed according to the uniform distribution: $x_i \sim U[0, 1]$, $i = 1, ..., N$. For simplicity, assume that the principal cares only

[12] However, mixed strategy implementation can be optimal if the agent's utility function has a strictly convex part. Examples can easily be constructed for the case where there exists a minimum utility level so that $u(x) \to u_{min}$ for $x \to -\infty$, or when the utility function is convex in losses and concave in gains (e.g., as in prospect theory (Kahneman and Tversky 1979)).

about the sum of outputs from these tasks in the first period:

$$\tilde{Y} = \sum_{i=1}^{N} \tilde{y}_i. \tag{18}$$

For illustrative purposes, we will restrict attention to the simple case where the agent can exert effort only in a single task. If the principal wants to induce effort in a specific task i, she offers a contract that defines a *clear mission* by implementing pure strategy effort in this *specific* task. That is, transfers would be $t_1(\underline{y}_j) = t_1(\bar{y}_j) = 0$ for $j \in \{1, ..., N\}$, $\jmath \neq i$, $t_1(\underline{y}_i) = 0$ and $t_1(\bar{y}_i) = g\left(\frac{\psi}{\pi}\right)$. In contrast, if the principal wants to create ambiguity about the agent's human capital she offers an n-task *fuzzy mission* contract that rewards the agent based on the sum of outputs over $n \leq N$ tasks. For example, if the principal wants to implement effort in one of the first n tasks in the limited liability case, transfers would be $t_1 = \begin{cases} g\left(\frac{\psi}{\pi}\right) & \text{if } \sum_{i=1}^{n} y_i = \bar{y} \\ 0 & \text{if } \sum_{i=1}^{n} y_i = 0 \end{cases}$. Under such a contract the market only imperfectly learns whether effort was exerted on a specific task or not. If it observes $y_i = \bar{y}$ for some task i, it is clear that the agent exerted effort on this task and acquired human capital $\tilde{H}_i$. Thus, ex ante expected second-period human capital in the case of high output is $\bar{H}/2$. In contrast, if the market observes $y_i = 0$, for all $i = 1, ..., N$, the market does not know which task was actually pursued and attributes probability $1/n$ to each task $i = 1, ..., n$ and probability zero to each task $i = n+1, ..., N$. The agent's best wage offer thus will be

$$\max_{i \in \{1,...,n\}} \frac{\tilde{H}_i}{n}. \tag{19}$$

Taking expectations, ex ante expected second-period human capital in the case of low output is $\underline{t}_2(n) = \frac{n}{n+1} \frac{\bar{H}}{n}$.[13] Thus, under an n-task fuzzy mission contract the wedge between high and low output states in terms of expected reputation is given by $\frac{(n-1)}{2(n+1)} \bar{H}$. Since the expected output for the principal under both types of contracts is the same, and the reputational wedge is increasing in n, it is optimal for her to set $n = N$. Thus, it is optimal for the principal to give the agent complete autonomy of decision over the production process.

Note that in this simple example focus does not matter for production and therefore it is optimal to maximize reputational incentives. The framework can easily be extended to allow for firm-specific human capital and productive gains from focus. Nevertheless, the finding that fuzzy mission contracts can dominate those that implement a clear mission is interesting, since it reverses the result that obtains in the pure career concerns setting of Dewatripont,

[13] This uses the fact that the k-th order statistic in a sample of n observations of the uniform distribution on $[0, 1]$ follows a Beta distribution with parameters k and $n - k + 1$.

Jewitt, and Tirole (1999b), in which there is no learning by doing. In their model, under some regularity conditions, a principal always prefers clear missions.

5.2 Screening of Job Seekers

In this section we address the issue of optimal screening of job seekers by extending the base model to allow for ex ante uncertainty about agents' types. In the first period, the principal has a vacant position to be filled and can hire from a pool of unexperienced job seekers. These start off with an initial level of human capital of zero and are protected by limited liability. Once hired by the firm, an agent who does not exert effort produces low output $\underline{y}$ and acquires human capital $\underline{H}$ in the first period. A proportion λ of job seekers is *high-skilled*, and can additionally exert unobservable effort at a private cost ψ to increase their unobservable human capital from $\underline{H}$ to $\bar{H}$ through learning by doing. In that case, they produce high output $\bar{y}$ with probability π. The firm has access to a screening technology which provides an informative signal regarding the types of job applicants with probability q. To set the most favorable conditions for perfect screening, assume that the principal can choose any probability $q \in [0, 1]$ at no cost. The choice of screening precision is observable by the market.
If the principal adopts the perfect screening technology $q = 1$, she can distinguish job seekers' abilities and hires a high-skilled agent. In equilibrium, the market correctly anticipates this hiring decision and the incentive problem corresponds exactly to the one in section 3.1. Since the agent is protected by limited liability, he receives a transfer $g(\frac{\psi}{\pi})$ if he produces high output and 0 otherwise. In contrast, if the principal adopts an imperfect screening technology with $q < 1$, she ends up hiring a high-skilled agent with probability $q + \lambda(1 - q)$ and a low-skilled agent with probability $(1 - \lambda)(1 - q)$. Then, high output provides an agent with a marketable signal that reveals him to be of high ability and increases his second-period earnings compared to the situation in which he produces low output and his human capital is uncertain. The derivation of the optimal screening precision q is similar to the analysis in section 3.2, replacing p by $q + \lambda(1 - q)$. Thus, we obtain the following corollary to proposition 1:

Corollary 1

If agents are protected by limited liability and the principal has access to a costless screening technology, then a sufficient condition for imperfect screening (screening precision $q < 1$) is given by condition (10), replacing p by $q + \lambda(1 - q)$.

Our result that a firm might choose not to perfectly screen job seekers even if perfect screening is costless is reminiscent of Crémer (1995). He also shows that a principal might optimally choose not to acquire information about an agent. The rationale is however different from ours. In Crémer (1995), if the principal were to acquire information about the agent she would choose to renegotiate and retain an agent who turns out to be of high ability, despite low performance. Anticipating this, high-skilled agents would exert no effort. Thus, choosing to remain ignorant about an agent's type allows the principal to credibly dissociate incentives for effort from the desire to retain high-skilled agents. In our model, the principal limits the amount of information that she acquires for a different reason. Committing to an imperfect screening technology, $q < 1$, allows her to make the beliefs of the market regarding the agent's human capital react to the output produced. This provides a high-skilled agent with reputational incentives and permits the principal to cut back on monetary incentives for effort.

Another interesting implication of our result is that the contracts offered by the firm in the first period are not type contingent. When screening is imperfect, reputational incentives arise only because contracts do not resolve the ambiguity about the agent's type. This provides a rationale for the phenomenon that wages often are less sensitive to differences in individual characteristics than predicted by incentive theory (e.g., Baker, Gibbs, and Holmström (1994)).

6 Conclusion

We show that in a sequential contracting model with moral hazard and learning by doing, a principal can benefit from implementing mixed strategy effort rather than pure strategy effort when agents are protected by limited liability. Mixed strategy effort provision gives rise to reputational incentives that lower the implementation cost for the principal. If these savings exceed the expected loss in output due to lower effort, the principal implements a contract that induces mixed strategy effort provision. Moreover, we demonstrate how our base model can be used as a building block to analyze issues such as mission focus or optimal screening of job applicants. In these extensions we show that it may be optimal for a principal to be vague on the type of task that an agent should pursue and delegate completely this decision to the worker. Furthermore, we derive conditions under which a principal refrains from screening heterogeneous job applicants ex ante, even if perfect screening is costless.

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