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The Effects of Remedial Mathematics on the Learning of Economics: A Natural Experiment*

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Abstract

This paper examines the effects of remedial mathematics on performance in university-level economics courses using a natural experiment. We study exam results prior and subsequent to the implementation of a remedial mathematics course that was compulsory for a sub-set of students and unavailable for the others, controlling for background variables. We find that, consistent with previous studies, the level of and performance in secondary-school mathematics has strong predictive power on students' performance at university-level economics. However, the evidence for a positive effect of remedial mathematics on student performance is relatively weak and is limited to a few sub-groups of students.

Keywords: remedial mathematics, teaching of economics, difference-in-differences, heterogeneous treatment effects, quantile regressions

JEL codes: A22 (Economics Education and Teaching of Economics—Undergraduate), I20 (Education—General)

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Introduction

The mathematical demands of university-level economics make it a difficult subject for less technically able students. A number of studies have shown a strong link between mathematics background and performance in economics degrees—see, e.g., Reid (1983), Anderson et al. (1994), Butler et al. (1994), Durden and Ellis (1995) and Lopus (1997). However, the nature of this relationship is not fully understood. Most studies use single measures of mathematics capability such as the American SAT score, which can not distinguish between students' inherent ability in maths and the level to which they studied the discipline prior to starting university. The latter possibility suggests that the implementation of remedial programmes in mathematics may be an important tool in helping students to cope with the demands of university-level economics.

In the United States, the use of remedial education at the university level has been extensive and controversial. According to data from the National Center for Education Statistics data, in 2000 71 per cent of all two- and four-year degree granting institutions offered remedial courses in mathematics. These courses were taken by 22 per cent of entering first-year students.[1] Proponents have argued that the programmes can help less technically able students, who are often from disadvantaged backgrounds, and facilitate their integration into university studies.[2] Critics have argued that remedial programmes are too costly and that tax dollars should not be used in colleges to teach high school courses. In the past decade critics of remedial

[1] The White House Social Statistics Briefing Room, http://nces.ed.gov/ssbr/pages/remedialed.asp?IndID=16 (accessed 11 July 2007).

[2] For accounts of these discussions, see e.g. Merisotis and Phipps (2000) and Bettinger and Long (2006).

programmes have been increasingly influential in public policy debates, and public universities in a number of states (including New York, California, Illinois, and Florida) have reduced students' access to remediation or eliminated their programmes entirely (Merisotis and Phipps (2000) and Bettinger and Long (2006)).

In light of the debate on remedial education, it is important to understand whether and to what extent it actually works. This paper examines the effects of remedial mathematics instruction on students' performance in economics subjects in a British context. We make use of a natural experiment created by the implementation of a remedial maths course for a sub-set of students in the economics programme at Royal Holloway, University of London. Prior to 1999 the Department of Economics did not offer remedial maths to any students. Beginning in 1999 the remedial "Foundations of Mathematics" (henceforth FoM) became compulsory for all single and joint honours students who either did not take A-level mathematics or received a grade C or lower at A-level mathematics.[3] The subject covered a sub-set of the A-level syllabus and was designed to ensure that all students had a basic grounding in the most important mathematical techniques used in an undergraduate economics degree.

We use administrative records of students entering the Department in 1997-1999 to examine the effects of the implementation of FoM on performance in first-year subjects.[4] The data covers individual characteristics, performance at A-level and exam

[3] An A-level (short for "Advanced level") is a qualification that can be taken by students in England, Wales and Northern Ireland, usually in the final two years of their secondary education (at the age of 16 to 18). They are available in a wide range of subjects. The number of A-level exams students take vary, but the minimum number required for university entrance is typically three. A-levels are graded A, B, C, D, E, N and U, with A being the top grade and where N ("nearly passing") and U ("Unclassified") are fail grades.

[4] We do not use the cohorts of students who entered in 2000 or later in our analysis because of a change in College policy that year that reduced the weight of the first year in the overall degree classification

results. We examine students' grades in compulsory first-year subjects ranging from the highly mathematical Quantitative Methods to the non-mathematical Economics Workshop. As with FoM, these subjects were either compulsory or unavailable to all students in the data set, depending upon their degree programme. We analyse student performance in these classes, examining the results prior and subsequent to the implementation of FoM using a difference-in-differences approach.

The relatively few previous studies on the topic have found that, in an American context, remedial mathematics seems to work. Johnson and Kuennen (2002) examine the scores of students in introductory microeconomics at a midwestern American university. They find that students who were assigned to remedial maths and actually took the course prior or concurrent to taking microeconomics performed better than students who were assigned to remedial maths but who waited until after taking microeconomics to take the remedial course. This approach leaves open the possibility that Johnson and Kuennen's results are driven by unobserved heterogeneity in the sample—e.g., that students who chose to take the remedial maths course relatively soon after having been assigned to it were on average more motivated than students who waited to take the course. Bettinger and Long (2006) examine remediation in the Ohio public university system using an instrumental variables technique to control for individual-level heterogeneity. They instrument having actually received remedial education with the estimated probability of receiving remediation given students' entry scores and exogenous variation in the choice of school attended, based on distance from home. Their findings indicate that students with a higher probability of being exposed to remediation are *ceteris paribus*

from 16.67 per cent to zero per cent. We feel that the effect of this change in policy on students' incentives is sufficiently large to make any comparisons to earlier years meaningless.

less likely to drop out of college and more likely to transfer to a higher-level college and to complete a bachelor's degree. Although Bettinger and Long (2006) are careful to construct a measure of remediation that is unlikely to be related to individuals' ability or motivation, there remains a couple of problems that make it difficult to interpret their results as a pure effect of remedial maths on students' performance. First, their instrument for remediation is based on school-level differences in remedial offerings, which could be correlated with other aspects of the schools' focus on undergraduate education (e.g. availability of tutoring, level of guidance etc). Secondly, they find that students of a given ability who had expressed an interest in majoring in a mathematically-oriented subject prior to entering college and who were assigned to take remedial maths were more likely to graduate, but less likely to do their degree in a mathematically-oriented subject than those not exposed to remediation. This suggests that remediation may have a limited effect in increasing students' mathematical abilities, but helps to match students with appropriate degree subjects.[5]

The design of this study reduces the impact of these concerns. The Foundations of Maths programme was either compulsory or unavailable for all individuals in the data set. Moreover, students required to take FoM all had to take it at the same time and from the same set of lecturers. Similarly, most of the courses taken by the students in our sample were compulsory, as were the times and the lecturers. Thus, it is highly

[5] There also exist of number of papers that study the effect of remedial education programmes that (i) are aimed at school pupils rather than university students or (ii) do not concern maths or numerical skills but, for example, writing skills. Examples of such studies are Aiken et al. (1998), who evaluate the efficacy of a one-semester remedial English course at an American university; Banerjee et al. (2007), who study a programme aimed at helping Indian school children who lag behind in basic literacy and numeracy skills; Jacob and Lefgren (2004), who evaluate a remedial programme involving summer school and grade retention for children in public schools in Chicago; and Lavy and Schlosser (2005), who study the effects of a programme that offered additional instruction to high-school students in Israel.

unlikely that the results of this study are driven by issues of selection bias created by unobserved heterogeneity across courses or students. In addition, the students in our sample were required to choose their subject of study prior to beginning their university studies, and could only change subjects by starting another degree from the beginning. Thus remediation is very unlikely to have an effect on matching students with appropriate degrees and would only be effective if it increases students' ability to complete their chosen degree.

The outline for the remainder of the paper is as follows. The first section describes the Royal Holloway economics programme and the natural experiment created by the implementation of Foundations of Mathematics. The second section describes the data set. The third section outlines our econometric methodology. The fourth section presents the estimation results for the range of compulsory economics courses and presents a variety of robustness checks of these results. The fifth section concludes. Our findings suggest that, consistent with previous studies, the amount of mathematics taken prior to university and the results in A-level mathematics have strong predictive power on student performance. However, we find relatively weak evidence that taking remedial mathematics has an effect on student performance. In regressions subdividing the students who were required to take remedial maths, we find, for one of the courses, a positive and statistically significant effect for the set of students who had not taken A-level maths. In addition, quantile regressions show a positive effect for quantiles at the upper part of the distribution.

1. The Royal Holloway Economics Degree and Foundations of Mathematics

The students contained in the data set were all enrolled in single honours economics, a major in economics, a joint degree in economics or a minor in economics at Royal Holloway, University of London. Once they had selected their degree programme, the set of subjects that students were required to sit in their first year was largely specified by the College's rules. All first-year single-honours students and majors were required to take Principles of Economics, Quantitative Methods I and Economics Workshop. Single-honours students also took a fourth unit, and had the option of choosing that unit in Economics or in another department. Joint-honours students were required to take Principles and Quantitative Methods I. Students with an economics minor were only required to take Principles. Joint students and economics minors took the remainder of their subjects in other departments, and thus did not have the option of taking additional economics courses. The department only ran a single lecture session of each course, which means that all students in a particular subject in a given year went to the same lectures and faced the same assessment.

The FoM module was created in 1999 specifically in response to the observation that students who lacked a solid mathematics background often struggled in the first year of their degree programme, particularly in Quantitative Methods I. Prior to the start of the first term, students were assigned to FoM if they had not taken or failed to attain at least grade B in A-level maths. The FoM module contained an hour of lecture and an hour of tutorial per week and ran concurrently to lectures in the Quantitative Methods I course over the first seven weeks of the first term. Students were not given choices of lecturers, seminar leader, or time of day when they took FoM. Because FoM ran parallel to the Quantitative Methods I course, it could not be taken by economics minors.

This set of rules provides an ideal experiment design that enables us to avoid the econometric problems potentially created by self-selection of students into courses based on factors such as mathematics ability. The choice of degree programme was normally made at the time of initial application to university, a year prior to the start of the degree. Although it is possible that students chose their degree programme partly based on mathematics ability, the extent of selection bias in this study is almost certainly less than for other studies in which students had options over which courses to take, and over the lecturer, time of day and point in their college career when to take a given course. Moreover, the Foundations of Mathematics course was made part of the curriculum only after the admissions process for 1999 was well under way and was not well publicized to applicants until the following year, and thus it is highly unlikely that it entered into the application decisions of prospective students.

The nature of FoM meant it was, at best, an imperfect substitute for A-level maths. The FoM module covered only a selection of topics from the full syllabus of A-level mathematics deemed most relevant to an economics degree, and ran over seven weeks, rather than the two years for A-level maths.[6] There was no examination or other assessment for FoM, which may have meant that students lacked incentives to put in much effort into FoM. However, the relevant comparison for this study is between students who have taken FoM and otherwise similar students who have not taken FoM, rather than between those with FoM and those with A-level maths. We

[6] The course covered basic algebra, functions, quadratic expressions, inequalities, logarithms and exponentials.

believe that, at a minimum, the implementation of FoM exposed students to more maths teaching than they otherwise would have received. Students who did not attend FoM lectures or tutorials potentially could have been penalized by failure for the entire Quantitative Methods I unit, which would result in an inability to progress to the second year of their degree. It is possible that a longer and more comprehensive remedial maths programme would be more effective than FoM; however, such a programme would also have higher costs in the form of additional teaching time, difficulties with scheduling alongside other courses and diverting students' time away from their other subjects.

2. The Data Set

The data set used in this study is constructed from administrative records kept by the Economics Department.[7] These records contain end-of-year exam results for all students entering in years 1997-99. We have linked these records to admissions records for students who took A-levels prior to entering University and who graduated in July 2000 or were still enrolled in the Department in August 2000. The admissions records contain information on age, sex, A-level results and results from all subjects taken at university. We do not have this data for students who began with the 1997, 1998 or 1999 cohorts and withdrew prior to August 2000, and thus we exclude these students from our analysis. The inclusion of only students who were present in August 2000 in the data means that our analysis only captures the effects of remediation on the group of students who are infra-marginal—at a comparative disadvantage in

[7]Royal Holloway, Department of Economics, UCAS Forms (various years); Royal Holloway, Department of Economics, Economics Sub-board: Provisional Rank Sorted Classification Grid (various years); Royal Holloway, Department of Economics, Annual Monitoring (various years).

mathematics, but nevertheless unlikely to drop out of university. Thus we cannot replicate Bettinger and Long's (2006) analysis on dropping out. However, in the European context where drop-out rates are much lower than in the US it seems more natural to focus on exam results.[8] Most withdrawals occurred early in the first year, well prior to students sitting any exams.[9] Anecdotally these withdrawals were more likely to be due to financial or personal reasons than due to difficulties with the subject.

For the purposes of our analysis the collection of data at a single survey date raises concerns about survival bias. The FoM programme may have had the effect of enhancing weaker students' performance enough so that they were able to stay in the programme, but nevertheless did not perform particularly well on their exams. This would lead to a downwards bias in the estimated effects of Foundations of Mathematics. We are not able to directly test for this effect in the data; however, we do note that aggregate statistics do not reveal a substantial change in the pattern of first-year withdrawals over the period of this study.[10] A second concern is that the full sample of students will not be comparable across years. Students entering in 1997 may have sat their first-year exams, but been excluded from the sample due to withdrawing during their second or third years at University. Similarly, students entering in 1998 may have withdrawn after their second year. Since we do not observe all students who entered in 1997 or 1998 and sat their first-year exams, we have

[8] In typical years over 80 per cent of students remaining in the programme until December of their first year went on to graduate. Royal Holloway, Annual Monitoring (various years).

[9] Our statistics on enrolments and drop-outs are very incomplete. For each cohort we have a count of all students (including those with entry qualifications other than A-levels) enrolled in January and those sitting end-of-year exams. Thus we do not observe the number of drop-outs during the first few months of the programme.

[10] The percentage of first-year students (including those with entry qualifications other than A-levels) dropping out between January of the first year and graduation was 8.9 per cent and 7.4 per cent respectively for 1998 and 1999 entrants. Royal Holloway, Annual Monitoring (various years). We do not have similar figures for 1997 entrants.

excluded from the sample all students who withdrew before sitting their final-year exams in order to ensure that the sample is comparable across years.

Table 1 provides descriptions of the main variables and summary statistics (means, standard deviations and the number of observations) for the sample as a whole and separated into four categories based on whether FoM would have been available and/or required.[11] The first four variables ($GRADE_m$, where $m=PE$, $QM1$, EW, EA) are our main dependent variables. The first three of these are the students' grades in the three courses mentioned previously. The fourth variable, $GRADE_{EA}$, is the students' overall average grade across first-year Economics courses. Note that $GRADE_{EA}$ is not simply a linear combination of the other variables, as the set of first-year courses can include additional optional courses other than the three previously mentioned. The scale used for the grade in all courses is the usual one, running between 0 and 100.

Next in Table 1, we summarize the main independent variables in the regressions. The variable *ALEVEL* is a summary measure of the students' A-level results. In coding this variable, we followed the standard approach for UK university admissions prior to 2002, summing up each student's points in their three best A-levels, where A = 10, B = 8 etc.[12] We have also separated out A-level results in mathematics and economics. The variable *FOM* is central to our analysis. It is the dummy variable that

[11] As the students' degree programmes differ and they accordingly did not all take the same courses, the number of observations is not the same for all variables. Table 1 reports the number of observations for students taking Principles of Economics, the largest course in the Department.
[12] We have followed standard admissions practices when coding the A-level scores for this paper. Thus an AS-level (advanced subsidiary) result is counted as half of an A-level result. We have also excluded A-levels that would not be counted for an admissions decision, i.e., general studies and native language. The alternative approach would be to use a set of dummy variables for different A-level results. However, this approach has a significant cost in terms of degrees of freedom and the number of observations in many of the cells would be very small. Thus the resulting regression estimates are likely to be imprecise.

gives us the difference-in-differences estimate. It takes the value one if a student entered in 1999 (the year FoM was introduced) and was assigned to FoM (i.e., if the student (i) had a grade C or lower in A-level maths or did not take A-level maths and (ii) was not an economics minor); otherwise it equals zero. For some of the regressions we also disaggregate students taking FoM into two and three groups, respectively, based on whether they took A-level maths and their grade in that subject. Finally Table 1 contains information on background variables such as gender, degree programme and year of entry.

3. The Econometric Model

Our econometric model postulates that student *i*'s grade in course *m*, $GRADE_{mi}$, is a linear function of mathematical ability, MAB_i, and "other skills", OSK_i:

$$GRADE_{mi} = \alpha_{0m} + \alpha_{1m}\, OSK_i + \alpha_{2m}\, MAB_i + \varepsilon_{mi}, \qquad (1)$$

where the ε_{mi}s are independent and normally distributed error terms. Although the variables MAB_i and OSK_i can not be observed, they are stochastically related to other variables that *are* observable. In particular we assume,

$$OSK_i = \beta_0 + \beta_1\, X_i + \beta_2\, Z_i + \omega_i \qquad (2)$$

and

$$MAB_i = \gamma_0 + \gamma_1\, X_i + \gamma_2\, Y_i + \delta_i\, FOM_i + \mu_i, \qquad (3)$$

where the ω_is and the μ_is are independent and normally distributed error terms. As (2) and (3) suggest, X_i is a column vector of explanatory variables that affect both OSK_i and MAB_i, Y_i is a column vector of explanatory variables that affect MAB_i but not OSK_i, and Z_i is a column vector of explanatory variables that affect OSK_i but not MAB_i. When fitting our regressions we will use several specifications, but throughout we will take Y_i to include (a sub-set of) the variables in the data set that indicate whether a student has taken A-level maths and the grade in that subject—i.e., *AMATH*, *AMATHA*, *AMATHB*, *AMATHC* and *AMATHDE*—and X_i and Z_i to include (a sub-set of) the remaining independent variables in the data set (overall A-level score, sex, cohort etc). The choice of which variables to include in Y_i is important for some of the statistical tests that we will conduct (see below). The αs, βs and γs are (vectors of) parameters that are assumed to be identical across individuals. The key parameter of interest, the scalar δ_i, may be individual specific, allowing for heterogeneous treatment effects.

Substituting (2) and (3) into (1) yields the following reduced form:

$$GRADE_{mi} = (\alpha_{0m} + \alpha_{1m}\beta_0 + \alpha_{2m}\gamma_0) + (\alpha_{1m}\beta_1 + \alpha_{2m}\gamma_1)X_i + \alpha_{2m}\gamma_2 Y_i$$

$$+ \alpha_{1m}\beta_2 Z_i + \alpha_{2m}\delta_i FOM_i + (\alpha_{1m}\omega_i + \alpha_{2m}\mu_i + \varepsilon_{mi})$$

$$= a_m + b_m X_i + c_m Y_i + d_m Z_i + f_{mi} FOM_i + e_{mi}. \qquad (4)$$

Our difference-in-difference methodology will be able to identify a certain average of the f_{mi}s, namely the average f_{mi} among those students who satisfy the criteria for being assigned to FoM, i.e., the average among the treated students; we denote this by f_m (= $\alpha_{2m}\delta$, where δ is the average of the δ_is among the treated). If we were to estimate one of the equations in (4) and found the estimate of f_m not to be significantly different from zero, then this could be: (i) because, on average among the treated students, FoM failed to have an effect on maths ability even though maths has a positive effect on the grade in course *m* ($\delta = 0$ and $\alpha_{2m} > 0$); (ii) because FoM did have a positive effect on maths ability but maths ability does not matter ($\delta > 0$ and $\alpha_{2m} = 0$); or (iii) because neither FoM nor maths ability matters ($\delta = 0$ and $\alpha_{2m} = 0$).

Given the data we have available, it is not possible to separately identify the parameters δ and α_{2m}—only their product f_m. We can, however, perform separate tests of the hypotheses that FoM failed and that maths does not matter in course *m*, respectively. First, if maths ability does not affect the score in course *m*, so that $\alpha_{2m} = 0$, then this implies $c_m=f_m=0$ (see equation (4)). For each of the four courses we can, using an F-statistic, test this null hypothesis against the alternative hypothesis that $c_m > 0$ or $f_m > 0$ (or both).

Hypothesis 1: *Mathematics ability has no effect on the grade in course m,* $c_m=f_m=0$.

Second, if FoM has no effect on maths ability, so that $\delta = 0$, then this implies $f_m= 0$ for all four courses (recall that $f_m = \alpha_{2m}\delta$).

Hypothesis 2: *Foundations of Mathematics has no effect on mathematics ability,* $f_{PE} = f_{QM1} = f_{WS} = f_{EA} = 0$.

4. Empirical Results

Table 2 reports single-equation OLS regressions with $GRADE_m$ as the dependent variable. Whereas Specification 1 is relatively parsimonious (measuring the students' A-level mathematics and economics background by the dummy variables *AMATH* and *AECON*), Specification 2 is richer (using dummies for the different grades at A-level maths and economics). The regressions do not provide particularly strong evidence that Foundations of Mathematics was successful in raising the students' grades. The estimated coefficient for *FOM* is significant at the five per cent level or better for only two of the equations—the equations with $GRADE_{WS}$ as the dependent variable. As we argue below, however, this effect is likely to be spurious, as the Workshop course is essentially non-mathematical and maths ability does not matter for being successful in it. One of the remaining *FOM* coefficients (Economics Average, Spec. 2) is significant at the ten per cent level. We do not rule out that this result could reflect a real effect of the remedial course on the students' overall average in first-year economics courses. However, it is also clear that the regressions reported in Table 2 do not provide a consistent story of success for the remedial programme.

It is surprising that the only *FOM* coefficient that is significant at the five per cent level or better is the one for Economics Workshop, given that this course is essentially non-mathematical with assessments only by submitted essays. Indeed the hypothesis that maths ability has no effect on the grade in Economics Workshop ($c_{EW} = f_{EW} = 0$) can not be rejected at the five per cent level for either one of the two specifications.[13] The corresponding hypotheses for the three other courses *can*, for both specifications, all be rejected at the one per cent level.[14] We have also estimated the Specification 1 and 2 versions, respectively, of the equations in (4) jointly. The qualitative features of the results are similar to those of the single-equation results found in Table 2, and thus

[13] Inspecting Table 2 we can observe that the model does a much worse job in explaining variation in the Economics Workshop grade than for the other course grades, with an R^2 value (Specification 2 regressions) of only 11.57 per cent, compared to between 27.02 and 49.44 per cent for the other courses. In addition, far fewer coefficients in the Economics Workshop regression are statistically significant compared to the regressions for the other courses. One possible explanation for these findings would be that the particular set of explanatory variables that we have available—which tend to have a focus on mathematics—are better at predicting grades in quantitative subjects. Another possible explanation would be that the results in an essay-based course like Economics Workshop are inherently more difficult to predict.

[14] That is, the tests that we perform are, for Specification 1, whether the coefficients for *AMATH* and *FOM* are both zero; and, for Specification 2, whether the coefficients for *AMATHA*, *AMATHB*, *AMATHC*, *AMATHDE* and *FOM* are all zero. For Principals of Economics the F-statistics (for Specification 1 and 2, respectively) are $F(2, 184) = 8.88$ and $F(5, 178) = 6.53$; for Quantitative Methods the F-statistics are $F(2, 172) = 49.25$ and $F(5, 166) = 22.56$; for Economics Workshop the F-statistics are $F(2, 146) = 2.88$ and $F(5, 140) = 1.36$; and for Economics Average the F-statistics are $F(2, 185) = 20.70$ and $F(5, 179) = 10.25$.

we do not discuss or report them separately. However, these estimates and the relevant F-statistics can be used to test Hypothesis 2 ("Foundations of Mathematics has no effect on mathematics ability"). As one might suspect by simply inspecting the relevant t-values in Table 2, this hypothesis can not be rejected at any reasonable level.[15]

Another striking observation from Table 2 is that, although all the grades that we try to explain are in economics courses, most of the A-level economics variables are not statistically significant, whereas most A-level mathematics variables are. In particular, for Specification 1, the *AECON* coefficient is never statistically significant whereas the *AMATH* coefficient is always significant at the one per cent level for all equations except Economics Workshop (in which, as noted above, almost no variable is significant at any level). This broad pattern—with the maths coefficients being more significant than the economics coefficients—is very clear also for Specification 2.[16] The general result that a strong mathematics background is a good predictor of performance in undergraduate economics courses is consistent with earlier work (see, e.g., Reid (1983), Anderson et al. (1994), Butler et al. (1994), Durden and Ellis (1995) and Lopus (1997)).

There are two possible interpretations of our finding that there is no, or only weak, evidence that taking remedial mathematics has an effect on student performance, and

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[15] That is, we test whether the coefficients for *FOM* in all four equations are zero. The F-statistics for Specification 1 is F(4, 576) = 1.70, and for Specification 2 it is F(4, 552) = 1.45.

[16] The Specification 2 regressions also reveal that the *AMATHA* coefficients typically are smaller in magnitude than the coefficients for *AMATHB* and *AMATHC*. This counterintuitive result appears to be at least partially due to the fact that the variable *ALEVEL*, which assigns linearly decreasing weights to the different grades, is included in the regressions. Dropping *ALEVEL* from the equations yields *AMATHA* coefficients that are more similar in magnitude to the ones for *AMATHB* and *AMATHC*. Moreover, for all four equations with a dropped *ALEVEL* variable, the hypothesis that the coefficients for *AMATHA*, *AMATHB* and *AMATHC* are all the same can not be rejected at any reasonable level.

we can not fully distinguish between these explanations. The first, and most obvious, explanation is that the existing Foundations of Mathematics programme did little to affect student ability in mathematics. The second explanation is that the remedial programme actually did have a positive impact on the students' maths skills, although the students, realizing that this improved ability would make it easier for them to achieve a particular grade in the other subjects, chose to substitute time and effort from the their studies in these other subjects to non-academic activities. Given that we do not have access to data on the amount of time the students spent on their studies, we can not rule out the second explanation.[17] Still, even under this scenario, the conclusion that introducing a remedial maths programme may not be helpful in improving performance in technically demanding economics subjects would still be valid, although for another reason than one perhaps first would think.

Another possible criticism of the results reported in Table 2 is that they strictly reflect short-term outcomes. It is possible that remedial maths increased students' mathematical abilities, and that this did not immediately translate into better exam results but nevertheless had an effect on longer term educational outcomes.[18] We have tested this hypothesis by looking at the effects of FoM on the performance in core second- and third-year subjects. In particular, we ran OLS regressions for grade in

[17] Dolton, Klein and Weir (1994) offer some evidence that students who are provided with improved teaching resources substitute time away from self-studies. The authors study a group of undergraduate first-year economics students who were offered assistance in their computer training from paid peer-counsellors. Compared to a control group who did not have access to counsellors, these students spent less time working with computers in classes and in preparing assessed work.

[18] This long-term effect could occur through two different channels: (1) While taking the remedial course, the students might be handicapped by the fact that the FoM module takes time away from their regular courses. Therefore, even if they do learn maths by taking FoM, this may not compensate for the associated loss in time, making the net benefit zero or even negative. In their second year, however, the remedial students do not have to spend time on FoM but may still benefit from any maths knowledge they have acquired by taking it in the previous year. (2) FoM may have an indirect effect in that it facilitates learning of the material in the first-year course Quantitative Methods I, which in turn helps the students to get a good grade in one or more second- and third-year courses like, for example, Quantitative Methods II.

Microeconomics (core for all second-year economics students), Macroeconomics (core for all second-year single honours, major and joint students), Quantitative Methods II (core for second-year single honours and majors) and the final-year dissertation (core for all third-year economics students). The qualitative results of these additional regressions are very similar to those of Tables 2, and they are therefore not reported.[19] Most importantly, *FOM* is non-significant in all specifications (in many cases with the "wrong" sign). Moreover, A-level maths is statistically significant in the specifications analogous to those in Table 2.

Recall that the students who were assigned to FoM were those who had taken A-level maths but got a grade of C or worse and those who had not taken A-level maths at all. Therefore, our estimates of the treatment effect in all the regressions reported in Table 2 refer to the average effect in a fairly large and heterogeneous group of students. It seems plausible that a remedial student's capacity for benefiting from FoM might depend on his or her maths background: whether the student had taken A-level maths or not, and, in the former case, the exact grade. To explore this, we run regressions that are analogous to the Spec. 2 versions of the regressions in Table 2 but with the remedial students divided into sub-groups according to their maths background. These regressions are reported in Table 3.

The first four columns of Table 3 show OLS regressions with *two* separate treatment groups: remedial students with bad A-level maths (*FoM_Amath*) versus remedial students with no A-level maths (*FoM_No_Amath*). With two exceptions, all treatment coefficients for these columns are non-significant (sometimes with the wrong sign).

[19] A full set of results is available on request from the authors.

The first exception is the Workshop course, for which the coefficients for both sub-groups are indeed significant and with the expected sign; however, exactly as with the results in Table 2, the A-level maths variables in the Workshop regression are in all cases non-significant, which casts doubt over the possibility that the estimated coefficients capture a real effect of FoM on the students' performance in the Workshop course. The second exception is Economics Average. While the treatment coefficient for the students with A-level maths is non-significant for this course, the coefficient for the students without A-level maths is indeed significant (and with the right sign) at the five per cent level. The point estimate for this coefficient is quite large in magnitude, suggesting that a student without A-level maths on average gains more than six marks by taking the remedial course. The next four columns of Table 3 show regressions with *three* separate treatment groups: remedial students with an A-level maths grade C (*FoM_AmathC*), remedial students with A-level maths grades D or E (*FoM_AmathDE*), and remedial students with no A-level maths (*FoM_No_Amath*). The results here are qualitatively the same as for the regressions with two treatment groups. In particular, in the Economics Average regression the coefficient for the students without A-level maths is again significant at the five per cent level. The other two treatment coefficients are not significant, which suggests that distinguishing between students with grade C and D/E is not important.

In summary, the results in Table 3 are fairly close to the ones obtained when not distinguishing between different categories of remedial students. As before, we find a statistically significant (now at the five percent level) treatment coefficient for Economics Average. However, for a mathematically demanding subject like Quantitative Methods I, where we *a priori* would expect to find an effect, no treatment coefficient is significant. Yet when we do find an effect for some category of remedial students, it is consistently (except for the probably spurious effect for the Workshop course) for the group of students who did not do A-level maths at all, a result which strikes us as plausible.[20]

An interesting possibility is that FoM may have had an impact on the performance of students at non-central parts of the ability distribution, which would not necessarily be captured by the OLS regressions reported in Table 2. We explore this possibility by running quantile regressions with the scores in our first-year courses as dependent variables.[21] Such regressions—for the 0.25, the 0.50 and the 0.75 quantiles—are reported in Table 4 (all right-hand side variables are as in the Spec. 2 versions of the regressions in Table 2). For the first two quantiles, the *FOM* coefficient is not significant at any reasonable level. However, for the 0.75 quantile it is significant at the five and ten per cent levels, respectively, for the Workshop course and Economics Average. Again, we are sceptical about the relevance of the significant Workshop coefficient. However, the statistical significance of the coefficient for Economics Average is interesting. It suggests that FoM may have an effect on the performance of students who have a higher-than-average ability, although the remedial programme is not able to help students who are at the lower and middle parts of the ability distribution. Indeed, for all four subjects except Principles the *FOM* coefficients tend to be larger and more significant the larger are the quantiles.

[20] One possible interpretation would be that those students who took A-level maths but got a bad grade have already been exposed to maths teaching but proved to lack the ability or motivation to learn, whereas the students who never took A-level maths are on average more receptive to the teaching of basic maths. Another interpretation would be that the A-level maths students have—in spite of their bad grades—already learned the basic material taught in FoM and have nothing to gain from it.

[21] For other studies of the determinants of academic performance using the approach of quantile regressions, see e.g. Eide and Showalter (1998) and Dolado and Morales (2007). On quantile regressions in general, see e.g. Koenker and Hallock (2001) and Koenker (2005).

Figure 1 shows the results for a range of further quantile regressions, which reinforce the above conclusions.[22] The figure has three panels, for Principles of Economics, Quantitative Methods and Economics Average. Each panel depicts point estimates of the *FOM* coefficient (the connected dots) and 90 per cent confidence intervals (the grey area above and below the connected dots) for the quantiles in the range 0.25 to 0.75. In addition, each panel shows an OLS point estimate (the black horizontal solid line) and the associated 90 per cent confidence interval (the two horizontal dashed lines). Another (thicker) black horizontal line indicates the zero level. We see that the OLS estimate is not close to being statistically significant for the first two courses and that for Economics Average the lower boundary of the 90 per cent confidence band more or less coincides with the zero line (both observations being consistent with the results in Table 2). However, for all three courses the estimates obtained with the quantile regressions are, at least for a range of quantiles, statistically significant or close to being so. For Principles, the 90 per cent confidence band lies almost completely above the zero line for all quantiles in, approximately, the range 0.4 to 0.6; for Quantitative Methods I, the lower boundary of this band lies just below the zero line for the quantiles from, approximately, 0.55 and upwards; and for Economics Average the band lies clearly above the zero line for the quantiles from the median and upwards.[23]

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[22] The additional regressions and the figure were produced using the free software R (http://www.R-project.org).

[23] We have also examined similar figures for a sub-set of second- and third-year courses and for the same set of first-year courses as above but with two separate treatment groups. These figures indicate that the effect on students in second- and third-year courses is weak or non-existing (which is consistent with our OLS results), and for the first-year courses there is no discernable systematic difference between the effect on remedial students with and without A-level maths (which is in contrast to some of the OLS results reported in Table 3).

5. Concluding Remarks

The effectiveness of remedial programmes has important policy implications. The British Government has targeted expansion of participation in Higher Education from about 43 per cent in 2002 to 50 per cent by the end of the decade, while at the same time raising tuition fees for British and EU students from approximately £1100 to £3000. This increase in costs makes it likely that much of the demand for new university places will come in subject areas that offer good future earnings prospects, such as economics.[24] However, despite the earnings potential of an economics degree, recent growth in enrolment in economics programmes has lagged behind the overall growth of student numbers, perhaps partly due to the mathematical demands of the subject and recent declines in enrolment in A-level mathematics.[25] In this context, a successful remedial mathematics programme might not just improve the performance of currently enrolled students, it might also increase the demand to study the subject. In addition, successful remedial mathematics programmes may be an important tool to help meet the Government's Widening Participation targets.[26]

In light of the policy implications, it is perhaps surprising just how little evidence there is on the effectiveness of these programmes. This paper examines the impact of the implementation of a remedial mathematics programme for students' performance

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[24] Jessica Shepherd, "Careerist Mentality Rises with Top-ups", Times Higher Education Supplement, 17/2/2006, pp. 1, 4; also available at http://www.thes.co.uk/story.aspx?story_id=2027904 (accessed 11 July 2007). See Dolton and Makepeace (1990) and Bratti and Macini (2003) on the earnings of graduates in different subject areas.

[25] On enrolment in different degree subjects see Rebecca Smithers, "Maths 'crisis' as exam entrants tail off", Guardian Unlimited, http://education.guardian.co.uk/alevels2002/story/0,,774698,00.html and "Media studies overtakes physics", BBC News website, http://news.bbc.co.uk/1/hi/education/4162230.stm on the decline of enrolment in A-level maths (both accessed 11 July 2007).

[26] These targets involve making higher education more accessible to traditionally underrepresented groups, such as those from poor areas and weaker schools, mature students and students without standard A-level qualifications.

in a variety of compulsory economics courses at Royal Holloway, University of London. These courses differed considerably in their use of mathematics. For each course, we examined the results prior and subsequent to the implementation of the remedial mathematics course, using a difference-in-differences approach analysing student performance. In line with a large body of existing literature, we found that, for all courses except the least mathematically demanding one, the amount of mathematics taken prior to university and the results in secondary-school mathematics have strong predictive power on student performance. However, we find relatively weak evidence that taking remedial mathematics has an effect on student performance.

In our basic regressions we find little evidence that remedial mathematics has an effect on student performance. One exception is the essay-based course Economics Workshop, for which we find an effect that we argue is spurious, as there also is evidence that maths ability does not matter for performance in this subject. We also find an effect that is at the border of being statistically significant for the overall first-year Economics Average, an effect that becomes somewhat more significant when considering separately those students who did not take mathematics at A-level. It is important to note, however, that this result does not show up for all the subjects where one would expect an effect to be present. We also find that a series of quantile regressions provide some evidence that the remedial mathematics programme, although unable to help students who are at the lower and middle parts of the ability distribution, has a positive effect on the performance of students who have a higher-than-average ability. We obtain this result for most, although not all, of the first-year subjects; however, for most subjects and for most quantiles, the treatment coefficients are only at the border of being statistically significant at the ten per cent level. It therefore seems reasonable to conclude that, overall, the evidence that taking remedial mathematics has an effect on student performance is fairly weak. That said, we believe that the observation that there may be a positive effect of remedial maths for certain sub-groups of students is potentially important, and it would be useful if this possibility was investigated further in future research.

Our evidence also builds upon the existing literature on the effects of secondary-school mathematics education on university outcomes and on the effects of remediation. The result that secondary mathematics has strong predictive power is consistent with a wide body of empirical evidence. However, the result that remedial education in mathematics has little predictive power on performance in university-level economics is, to the best of our knowledge, new to the literature. We believe that there are two important distinctions between our study and the existing literature that has led to this result. First, our empirical framework does not suffer from problems of selection bias that occur in some other studies. Secondly, our outcome variable is course grades rather than graduation rates. On the one hand, it is possible that the focus on grades does not fully capture the effects of remediation on student retention. On the other hand, our results suggest that the effects of remediation on graduation rates shown by Bettinger and Long (2006) may reflect better matching between students and degree programmes, rather than improved performance in their originally chosen course.

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TABLE 1—VARIABLE DEFINITIONS AND SUMMARY STATISTICS

Variable	Description
$GRADE_{PE}$; $GRADE_{QM1}$; $GRADE_{EW}$; $GRADE_{EA}$	Grade in Principles of Economics, Quantitative Methods I, Economics Workshop, Average grade in all 1st year economics courses.
ALEVEL	Points on three best A-level subjects, excluding general studies, A=10, B=8, C=6, D=4, and E=2. (Can take values between 0 and 30.)
AMATH; AECON	Dummies: 1 if taking maths at A-level or AS-level; 1 if taking economics at A-level or AS-level.
AMATHA; AMATHB; AMATHC; AMATHDE;	Dummy: 1 grade in A-level math is A; B; C; D or E.
AECONA; AECONB; AECONC; AECONDE	Dummy: 1 grade in A-level economics is A; B; C; D or E.
JOINTMINOR	Dummy: 1 if taking joint degree or being an economics minor.
CLASS98; CLASS99	Dummy: 1 if entering in 1998; 1999.
FOM	Dummy: 1 if assigned to Foundations of Mathematics.

Variable	All Students	1997,98 entrants A-level Maths (A or B) or econ. minors	Other 1997,98 entrants	1999 entrants A-level Maths (A or B) or econ. minors	Other 1999 entrants
$GRADE_{PE}$	56.70 (14.54)	62.27 (15.05)	56.05 (12.78)	55.14 (11.09)	52.17 (18.07)
$GRADE_{QM1}$	57.70 (13.66)	66.62 (12.69)	52.17 (11.18)	68.75 (9.65)	57.89 (14.49)
$GRADE_{EW}$	60.07 (6.12)	60.00 (4.42)	58.94 (6.86)	59.29 (6.51)	63.22 (4.45)
$GRADE_{EA}$	57.59 (9.40)	61.76 (9.26)	55.45 (8.23)	57.37 (10.67)	58.07 (10.23)
ALEVEL	22.83 (3.30)	23.93 (3.36)	21.83 (3.28)	23.52 (2.44)	23.63 (3.02)
AMATH	0.62	1.00	0.39	1.00	0.51
AECON	0.71	0.61	0.80	0.67	0.66
AMATHA	0.09	0.24	0.00	0.29	0.00
AMATHB	0.25	0.74	0.00	0.67	0.00
AMATHC	0.22	0.02	0.29	0.05	0.37
AMATHDE	0.07	0.00	0.10	0.00	0.14
AECONA	0.27	0.30	0.26	0.24	0.26
AECONB	0.27	0.13	0.38	0.19	0.20
AECONC	0.13	0.13	0.14	0.10	0.14
AECONDE	0.05	0.04	0.02	0.14	0.06
MALE	0.65	0.72	0.61	0.71	0.60
JOINTMINOR	0.21	0.33	0.17	0.33	0.09
CLASS98	0.37	0.61	0.48	0.00	0.00
CLASS99	0.29	0.00	0.00	1.00	1.00
FOM	0.18	0.00	0.00	0.00	1.00
Sample Size	193	45	92	21	35

Note: Standard deviations shown in parentheses for continuous variables.

TABLE 2—SINGLE EQUATION REGRESSIONS, PERFORMANCE IN FIRST-YEAR COURSES

	Principles of Economics		Quantitative Methods I		Economics Workshop		Overall Economics Average	
	OLS ($GRADE_{PE}$)		OLS ($GRADE_{QM1}$)		OLS ($GRADE_{WS}$)		OLS ($GRADE_{EA}$)	
	Spec. 1	Spec. 2	Spec. 1	Spec. 2	Spec. 1	Spec. 2	Spec. 1	Spec. 2
CONSTANT	34.51 (4.47***)	31.91 (3.52***)	30.48 (4.89***)	35.33 (4.80***)	59.60 (15.98***)	58.96 (12.59***)	40.54 (8.72***)	42.30 (7.71***)
ALEVEL	0.90 (2.92***)	1.04 (2.72***)	0.90 (3.57***)	0.69 (2.19**)	-0.04 (0.28)	-0.02 (0.10)	0.65 (3.51***)	0.59 (2.53**)
AMATH	8.79 (4.07***)		15.96 (9.35***)		1.28 (1.23)		8.36 (6.43***)	
AMATHA		2.75 (0.67)		16.69 (4.78***)		2.99 (1.25)		7.54 (2.99***)
AMATHB		9.81 (3.54***)		18.14 (7.72***)		1.77 (1.18)		9.51 (5.57***)
AMATHC		12.88 (4.90***)		17.07 (8.18***)		1.05 (0.80)		9.51 (5.89***)
AMATHDE		0.25 (0.06)		8.44 (2.60***)		0.50 (0.23)		3.17 (1.25)
AECON	-1.24 (0.56)		-1.49 (0.82)		0.92 (0.83)		-0.79 (0.58)	
AECONA		0.29 (0.11)		0.69 (0.32)		0.36 (0.26)		0.54 (0.33)
AECONB		-2.83 (1.08)		-3.25 (1.50)		0.80 (0.58)		-1.88 (1.17)
AECONC		-2.71 (0.84)		-2.28 (0.85)		2.70 (1.58)		-1.91 0.96
AECONDE		15.53 (3.17***)		12.22 (2.84***)		1.56 (0.52)		8.21 (2.72***)
MALE	-0.89 (0.42)	-1.22 (0.61)	-0.85 (0.50)	-0.92 (0.57)	0.19 (0.18)	0.30 (0.28)	-0.70 (0.55)	-0.89 (0.72)
JOINTMINOR	-4.09 (1.65)	-4.63 (1.93*)	-2.94 (1.27)	-3.33 (1.49)	-2.21 (0.36)	-2.61 (0.42)	-5.25 (3.49***)	-5.72 (3.87***)
CLASS98	1.04 (0.44)	0.24 (0.11)	-2.79 (1.45)	-3.67 (1.98**)	-1.64 (1.38)	-1.79 (1.47)	-0.72 (0.50)	-1.32 (0.93)
CLASS99	-6.45 (1.79*)	-8.14 (2.20**)	2.46 (0.78)	-0.95 (0.29)	-1.34 (0.70)	-2.37 (1.11)	-4.05 (1.85*)	-5.85 (2.56**)
FOM	0.08 (0.02)	0.23 (0.05)	-2.90 (0.87)	0.56 (0.15)	4.53 (2.30**)	5.76 (2.35**)	3.30 (1.38)	4.88 (1.81*)
Observations	193	193	181	181	155	155	194	194
R^2	.154	.270	.420	.494	.097	.116	.258	.336
F	4.20***	4.71***	15.56***	11.59***	1.96*	1.31	8.04***	6.46***

Notes: Absolute values of t-statistics are shown in parentheses. * Statistically significant at the 10 per cent level. ** Statistically significant at the 5 per cent level. *** Statistically significant at the 1 per cent level.

TABLE 3—OLS REGRESSIONS, FIRST-YEAR COURSES WITH SEPARATE TREATMENT GROUPS

	Dependent variable							
	$GRADE_{PE}$	$GRADE_{QM1}$	$GRADE_{WS}$	$GRADE_{EA}$	$GRADE_{PE}$	$GRADE_{QM1}$	$GRADE_{WS}$	$GRADE_{EA}$
Selection of Indep. Variables	Two treatment groups: Bad (C/D/E) vs. no A-level maths				Three treatment groups: grade C vs. grade D/E vs. no A-level maths			
AMATHA	3.29 (0.80)	17.17 (4.89***)	2.93 (1.22)	7.86 (3.08***)	3.30 (0.80)	17.17 (4.88***)	2.93 (1.21)	7.86 (3.07***)
AMATHB	10.35 (3.65***)	18.66 (7.83***)	1.70 (1.11)	9.82 (5.63***)	10.35 (3.64***)	18.64 (7.81***)	1.70 (1.10)	9.82 (5.62***)
AMATHC	14.16 (4.76***)	18.42 (7.81***)	0.89 (0.58)	10.25 (5.62***)	13.76 (4.49***)	18.00 (7.38***)	0.81 (0.51)	10.00 (5.32***)
AMATHDE	1.70 (0.39)	9.94 (2.87***)	0.31 (0.13)	4.01 (1.48)	2.92 (0.59)	11.15 (2.87***)	0.62 (0.23)	4.76 (1.57)
FoM_Amath	-2.21 (0.43)	-2.08 (0.48)	6.02 (2.17**)	3.44 (1.09)				
FoM_No_Amath	2.57 (0.51)	2.92 (0.69)	5.47 (1.95*)	6.26 (2.01**)	2.55 (0.50)	2.91 (0.69)	5.46 (1.94*)	6.24 (2.00**)
FoM_AmathC					-1.03 (0.18)	-0.88 (0.19)	6.25 (2.11**)	4.18 (1.22)
FoM_AmathDE					-5.61 (0.69)	-5.45 (0.83)	5.31 (1.25)	1.33 (0.27)
FoM_No _AmathDE								
R^2	.274	.499	.116	.339	.275	.500	.116	.340
F	4.45***	10.95***	1.22	6.08***	4.17***	10.27***	1.14	5.69***
Observations	193	181	155	194	193	181	155	194

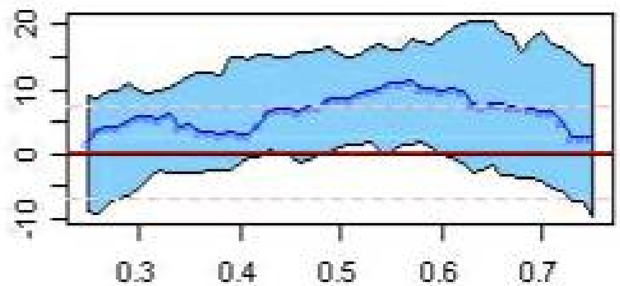
Notes: The base specification is the same as in the rich specification in Table 2, except that there are separate treatment groups as indicated. Absolute values of t-statistics in parentheses. * Statistically significant at the 10 per cent level. ** Statistically significant at the 5 per cent level. *** Statistically significant at the 1 per cent level.

TABLE 4—QUANTILE REGRESSIONS, FIRST-YEAR COURSES

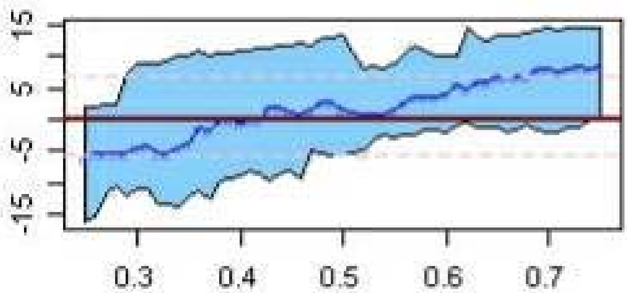
	Dependent variable											
	$GRADE_{PE}$	$GRADE_{QM1}$	$GRADE_{WS}$	$GRADE_{EA}$	$GRADE_{PE}$	$GRADE_{QM1}$	$GRADE_{WS}$	$GRADE_{EA}$	$GRADE_{PE}$	$GRADE_{QM1}$	$GRADE_{WS}$	$GRADE_{EA}$
Selection of Indep. Variables	The 0.25 quantile				The 0.50 quantile				The 0.75 quantile			
AMATHA	6.00 (1.00)	7.67 (1.13)	0.38 (0.14)	5.62 (1.44)	10.75 (1.92*)	17.25 (2.77***)	2.50 (1.14)	11.25 (2.88***)	5.05 (0.72)	23.67 (3.97***)	0.59 (0.34)	8.11 (1.86*)
AMATHB	11.00 (2.50**)	13.67 (2.26***)	0.17 (0.11)	6.55 (2.68***)	14.50 (3.44***)	22.06 (6.03***)	0.50 (0.35)	11.00 (3.91***)	14.29 (2.74***)	23.00 (9.13***)	0.96 (0.58)	10.54 (3.87***)
AMATHC	17.50 (5.18***)	16.11 (4.60***)	0.58 (0.39)	10.00 (6.71***)	15.00 (4.85***)	18.94 (7.59***)	0.50 (0.36)	9.40 (5.56***)	14.10 (3.23***)	17.33 (6.67***)	1.19 (0.82)	10.68 (4.34***)
AMATHDE	2.50 (0.39)	9.78 (2.03**)	1.75 (0.65)	1.87 (0.61)	0.75 (0.13)	8.31 (1.86*)	0.50 (0.28)	6.00 (2.02**)	2.57 (0.36)	9.00 (1.78*)	0.59 (0.33)	3.31 (0.95)
FoM	2.00 (0.31)	-6.33 (0.95)	4.42 (1.12)	-0.46 (0.12)	8.50 (1.45)	1.31 (0.23)	3.00 (1.31)	4.85 (1.34)	2.48 (0.32)	8.33 (1.56)	5.52 (2.06**)	7.37 (1.75*)
Pseudo R^2	.173	.298	.087	.262	.209	.346	.091	.226	.184	.368	.138	.213
Observations	193	181	155	194	193	181	155	194	193	181	155	194

Notes: The right-hand side variables are the same as in the rich specification in Table 2. Absolute values of t-statistics are shown in parentheses. Standard errors are based on 1000 bootstrap replications. * Statistically significant at the 10 per cent level. ** Statistically significant at the 5 per cent level. *** Statistically significant at the 1 per cent level.

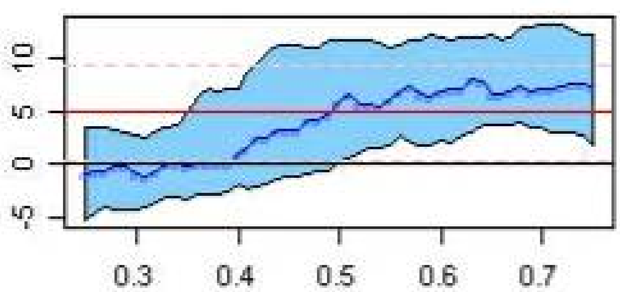
FIGURE 1—QUANTILE REGRESSIONS, FIRST-YEAR COURSES



Principles of Economics



Quantitative Methods I



Economics Average

Note: The right-hand side variables are the same as in the rich specification in Table 2.