Course content for MT1810, Number Systems

Prerequisites:
A-level Mathematics or equivalent

Aims:
This course aims to introduce fundamental algebraic structures used in subsequent courses and the notion of formal proofs, and to illustrate these concepts with examples.

Learning outcomes:
On completion of the course, students should be able to:

- apply Euclid's algorithm to find the greatest common divisor of two integers;
- use mathematical induction in a careful and logical way to prove simple results;
- perform arithmetic operations on complex numbers, using Cartesian and polar forms, locate points on the Argand diagram, and extract roots of complex numbers;
- prove De Morgan's laws and the distributive laws of set theory, and use the principle of inclusion/exclusion in simple cases;
- determine whether a given mapping is bijective and if so find its inverse;
- establish whether a given relation on a set is an equivalence relation and find the corresponding equivalence classes;
- compile truth tables to determine whether two statements are logically equivalent;
- define a ring, integral domain and field, establish some of their simple properties.

Course content:
The integers: division with quotient and remainder, binary numbers, the Euclidean algorithm, greatest common divisors, $\gcd(m,n)=sm+tn$, primes, statement of the fundamental theorem of arithmetic, the principle of mathematical induction, proof of the binomial theorem by induction.
Complex numbers: Cartesian addition and multiplication, the complex conjugate, rules of manipulation (the field axioms), inversion, the Argand diagram, modulus and argument, extraction of nth roots (quadratic equations and roots of unity (cyclic groups)), $\exp(it)=\cos t+i\sin t$, $\exp(z)$ and $\ln(z)$.
Sets: intersection, union, complement, Venn diagrams, De Morgan's laws.
Mappings: composition, associative law, injections, surjections, bijections and inverses. Equivalence relations and partitions. Propositional logic, truth tables.
Rings and fields: the ring Z_n of integers modulo n, the field Z_p . The ring $F[x]$ of polynomials over a field, analogy with Z (division law, monic polynomials, gcds), zeros, remainder and factor theorems, a polynomial of degree n over F has at most n zeros.