Course content for MT2320, Probability

Prerequisites:
MT1720 and MT1810

Aims:
To introduce the formalism of the mathematical theory of probability and thereby to lay a firm foundation for applications of probability in virtually all areas of science, including statistics, economics, the mathematics of financial markets, operational research, information theory, number theory, quantum theory and statistical physics.

Learning outcomes:
On completion of the course, the student should be able to

- demonstrate an understanding of the basic principles of the mathematical theory of probability;
- use the fundamental laws of probability to solve a range of problems;
- prove simple theorems involving both discrete and continuous random variables;
- explain the weak law of large numbers and the central limit theorem.

Course content:
Events and probabilities: Outcomes, events and probabilities. Probability spaces. Conditional probabilities. Independent events. The partition theorem. The Bayes Theorem.
Discrete random variables: Probability mass functions. Expectation. Multivariate discrete distributions. Independence. Expectation in the multivariate case. Indicator functions. Integer-valued random variables. Probability generating functions. Sums of random variables.
Distribution functions and density functions: Distribution functions. Continuous random variables. Functions of random variables. Expectations of continuous random variables. Random vectors and independence. Joint density functions. Marginal density functions and independence. Sums of continuous random variables. Changes of variables. Expectations of continuous random variables. Bivariate normal distribution.
Conditional expectation: Conditional probability mass functions and conditional probability densities. Conditional expectation as a random variable.
Moments and moment generating functions: Moments. Variance and covariance. Moment generating functions.
The main limit theorems: Chebyshev's inequality and the weak law of large numbers. The central limit theorem.