Course content for MT2720, Ordinary Differential Equations and Fourier Analysis

Prerequisites:
MT1710, MT1720 and MT1820

Aims:
This course aims to introduce the concepts of eigenvalues and eigenfunctions in the familiar situation of the trigonometric differential equation and to show how these yield Fourier series expansions for a general function. These Fourier series can be generalised to (a) generate more general eigenfunction expansions for a given function and (b) develop the Fourier transform, which is used in a variety of applications; its properties are investigated. The final step is to introduce a technique for solving differential equations where the coefficients are no longer constant.

Learning outcomes:
On completion of the course, students should be able to:

- locate eigenvalues both analytically and graphically;
- determine the Fourier series for a periodic function, including odd and even functions, and recognise the function represented by a given Fourier series;
- understand the role of eigenfunctions in building up a general function;
- orthogonalise a set of polynomials over a specified interval;
- manipulate the Dirac delta-function;
- manipulate and apply the Fourier transform;
- complete a solution-in series in straightforward cases.

Course content:
Introduction to Sturm-Liouville theory: eigenvalues and eigenfunctions; self-adjoint operators, orthogonal functions and their properties, orthogonalisation, completeness of eigenfunctions. Laguerre polynomials, Legendre polynomials.
Fourier series: Fourier-Euler formulae and statement of Fourier Theorem on the circle, Fourier sine and cosine formulae, extension to general analysis.
The Fourier transform: Fourier transform of derivatives, statement of Inversion Theorem, Dirac delta-function, Convolution Theorem, Parseval Theorem.
Ordinary differential equations: The Cauchy-Euler equation. Solution in series for a second-order linear differential equation, for two out of the four cases that can arise.