Course content for MT2900, Complex Variable

Prerequisites:
MT1710, MT1720 and MT1810

Aims:
This course is designed to provide an outline of the basic complex variable theory with some proofs. Applications are exhibited as used in other areas of mathematics. The object is to equip students to be able to use complex analysis to solve specific problems.

Learning outcomes:
On completion of the course, the students should be able to:

- use the definitions of continuity and differentiability of a complex valued function at a point, establish the necessity of the Cauchy-Riemann equations and apply this result;
- use a power series to define the complex exponential function and hence define the trigonometric and hyperbolic functions and the complex logarithm, and establish their properties;
- use the parametric definition of a contour integral in specific straightforward examples;
- state and use Cauchy’s Theorem, and apply Cauchy’s Integral Formulae to evaluate integrals;
- obtain Taylor series of rational and other functions of standard type;
- determine zeros and poles of given functions, and the residue at a simple pole and at higher order poles;
- state Cauchy’s Residue Theorem and apply it to evaluate real integrals (using Jordan's lemma when relevant) and to sum certain series, and state and use Rouché’s Theorem.

Course content:
Special functions: Power series and radius of convergence. Discussion of the exponential, trigonometric and hyperbolic functions for both real and complex variable. Definition of $\ln z$ and z^a.
Topology: An open (path-wise) connected set of the plane.
Functions of a complex variable: Continuity and differentiability of functions defined on an open set. The Cauchy-Riemann equations and Laplace's equation. Contour integrals along piecewise smooth curves C. Cauchy's theorem and Cauchy's integral formulae. Taylor series with examples, removable singularities, zeros and poles. Residue theorem and applications: calculation of simple integrals, including use of Jordan's lemma, and summation of infinite series. Principle of the argument. Rouché's theorem and the location of zeros of polynomials.